Submillimeter Array Technical Memorandum
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Performance of the Elevation Drive System
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Abstract This memo reports on measurements and modeling of the performance of the elevation drive. The best performance of the system is currently characterized by $3''$ steps. The fundamental problem appears to be excessive friction in the drive system. Either the friction needs to be reduced to about 1/3 its current minimum value, that is reduced to 20 ft lb from 60 ft lb, or the drive system must be fixed to handle a greater gain in the servo loop.

Introduction As soon as antenna 1 was contructed and put on its pad, the elevation drive was found to perform very poorly. The servo accuracy was found to be in arc minutes whereas the specifications called for sub-arc second accuracy. Movement of the dish was characterized by violent jerking and frequent shut down of the drive system due to excessive current draw. Measurements of the torque required to move the antenna showed a level of stiction (static friction or the torque required to initiate motion) of about 50-70 ft-lb minimum with the dish at its highest elevation where the dish is nearly balanced, increasing to more than one hundred ft-lb at the lowest elevation where the gravity load on the drive system is the highest. These measurements were made by converting the motor current to torque and later confirmed by moving the elevation drive manually with a torque wrench. The motor-screw-ball nut assembly tested on the bench with no load was found to have around 50 ft-lb of stiction implying that most of the stiction was in these components as opposed to the elevation axis bearings, for example. Further bench tests on the motor-screw-ball nut assembly determined that the ball nut bearing accounted for about half the stiction with the remainder in the motor assembly which includes other bearings and seals. The question then arises if the level of stiction/friction is too high, what level is permissable?

Previous technical memos (most of them without catalogued numbers) on the drive system offer little help principally because the drive system envisioned and discussed in these memos is not very much like the one actually built. Nor does the present drive system seem to have been discussed in any follow-up memos. For example, in the previous design memos, the friction was assumed to be negligible, less than 5 ft-lb, throughout the elevation drive. It is not clear that the present design could ever achieve that. The earlier memos assumed analog tachometers rather than digital encoders. At some later point, the hardware was changed again: the 16 bit encoders called for in the contract for the motor and current amplifier were replaced by 12 bit encoders. A 12 bit encoder has an accuracy of 1 arc second (after accounting for the 300:1 gear ratio) which is insufficient to measure velocity for a system tracking at a rate of a few arc seconds per second. The reason is that if the duty cycle is even as slow as 0.1 seconds, the smallest velocity that can be measured is $10''$/s.

The elevation drive needs to be redesigned in some way, and some hardware changes will be required. The purpose of this memo is to quantify in a rough way, the maximum level of stiction/friction in order to bound the elevation drive problem and help set the scope of required design work.

Characteristics of the drive system We determined the damping time and structural resonance of the dish by the following simple experiment. The motor was commanded to move one step, and the encoder readings of the motor and dish were recorded (figure 1).

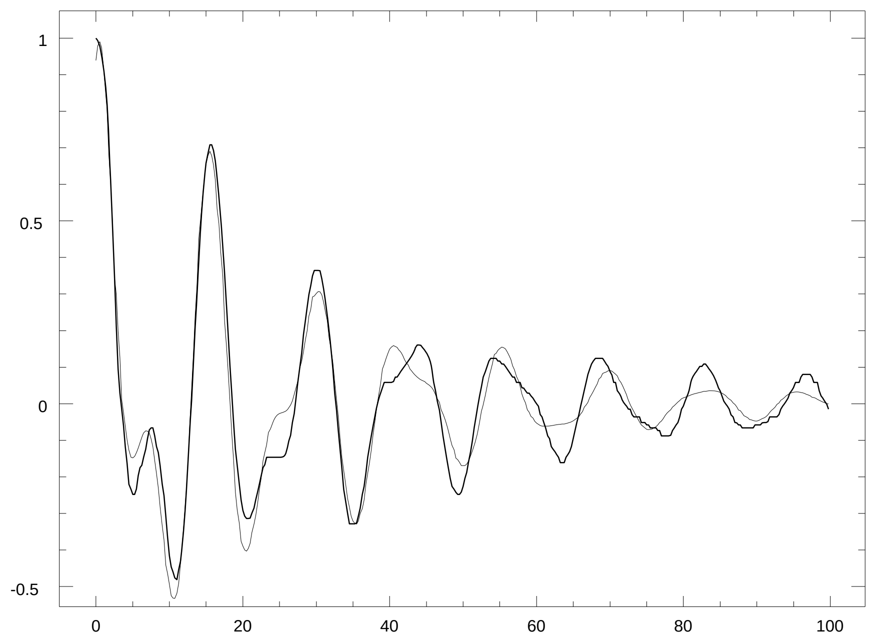

Figure 1. The heavy line represents the response of the dish to a sharp step by the motor. The vertical axis is the measured elevation axis encoder counts in arbitrary units, while the horizontal axis is time in hundredths of a second. The graph represents the movement of the dish in 1 second. The light line is a fit to the data based on a simple model of 2 masses connected by springs and dampers (Figure 2). From this data, we determine the characteristic frequency of the dish, 8.2 Hz, and the damping time of the oscillations, 0.5 seconds to damp to about 25% of the peak amplitude.

The data was analyzed by a simple model. We imagine the system to be composed of two masses, the dish and motor+screw connected by springs and dampers (figure 2).

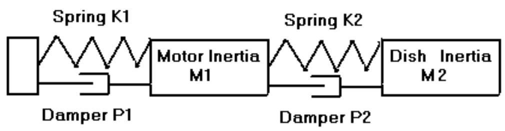

Figure 2. Model of elevation drive system with the motor operated as a stepping motor.

This system is described by a set of differential equations.

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ \dot{x_1} \\ x_2 \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-(k_1+k_2)}{m_1} & \frac{-(p_1+p_2)}{m_1} & \frac{k_2}{m_1} & \frac{p_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{p_2}{m_2} & \frac{-k_2}{m_2} & \frac{-p_2}{m_1} \end{pmatrix} \begin{pmatrix} x_1 \\ \dot{x_1} \\ x_2 \\ \dot{x_2} \end{pmatrix}$$

Here x_1 is the position of the motor and x_2 is the position of the dish. We then fit for the unknown parameters, the spring constants K_1, K_2, and the damping constants P_1, P_2. Using an inertia for the motor+screw of 0.886 kg m^2 and for the dish 29700 kg m^2, we find a spring constant for the motor, $K_1 = 4.1 \times 10^8$ Nm, and for the dish, $K_2 = 6.3 \times 10^7$ Nm, and a damping constant for the dish, $P_2 = 2.8 \times 10^5$. The damping constant for the motor is much less than for the dish, and the system damps through P_2. Thus P_1 is unimportant and cannot be determined from this experiment. From the derived constants we can determine the characteristic frequency of the dish as $1/2\pi\sqrt{K_2/M_2} = 8.2$ Hz and of the motor $1/2\pi\sqrt{K_1/M_1} = 11.4$.

Two minor points are worth mentioning. Because of the gear ratio at the ball nut, about 300:1, all the constants must be referred to one side of the gear ratio or the other. For example, the inertia of the motor+screw must be multiplied by 300^2 to obtain 80,000 kg m^2 to compare to the inertia of the dish. Thus the inertia of the screw is 3 times that of the dish. Finally, it should be noted that we do not actually know the inertia of the motor+screw and the dish for the following reasons. The screw has apparently been redesigned more than once and its diameter is variously listed in various documents. The adopted inertia is based on a diameter measured with a tape measure. The panels on the dish have been redesigned and are heavier than the original specifications.

Drive System Performance The servo system uses a computer designed for milling machines known as a PMAC. The computer has a built in so called Proportional-Derivative-Integral or PID servo control program which produces an output voltage which is a sum of 3 terms, one proportional to the tracking error, one proportional to the negative of the velocity, and one proportional to the time integrated tracking error. We can model the PMAC servo and antenna with our 2 mass model as a set of differential equations similar to those above. We also need to account for stiction and friction as additional and very important non-linear forces. In a usual case, we would measure the tracking error off the dish and the velocity off the motor.

$$\frac{d}{dt}\begin{pmatrix} x_I \\ x_1 \\ \dot{x_1} \\ x_2 \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{T} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{-g_I}{m_1 T} & \frac{-k_2}{m_1} & \frac{-(p_2+g_D)}{m_1} & \frac{k_2-g_P}{m_1} & \frac{p_2}{m_1} \\ 0 & 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{p_2}{m_2} & \frac{-k_2}{m_2} & \frac{-p_2}{m_1} & \end{pmatrix}\begin{pmatrix} x_I \\ x_1 \\ \dot{x_1} \\ x_2 \\ \dot{x_2} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_1} \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$$

Here, x_I is the integrated error term, $x_I = \frac{1}{T}\int_{t-T}^{t} x_2 dt'$, g_P, g_D, and g_I are the proportional, derivative, and integral gains, and F_1 and F_2 are other forces such as friction and wind. The friction force on the motor, F_1, is something like,

$$F_1 = \begin{Bmatrix} F_S & |v| < v_\epsilon \\ F_R & |v| \geq v_\epsilon \end{Bmatrix}$$

where F_S and F_R are values for static and running friction, and v_ϵ is some small velocity. In the model, F_2 is set to zero because the friction force acting on the elevation axis bearings is already included in the damping term P_2, and for the moment, we are not modeling the wind torque.

The PID control works very simply. The term proportional to the tracking error supplies a torque to reduce the error. The velocity or derivative term is meant to provide a damping force to reduce oscillations in a low friction system. Our system is highly damped by the excessive friction and the derivative term is not very important. The integral term is meant to take out systematic offsets as would be caused by a constant wind torque, for example. We have included the integral term in the equations, but not used the integral control in our antenna tests.

To control the antenna, we need to set a value for the proportional and derivative gains. Leaving out the integral term, the PD controller acting on an inertia or mass looks like a simple damped harmonic oscillator.

$$m\ddot{x} + g_D\dot{x} + g_P x = 0$$

As usual, the proportional gain, which is equivalent to a stiffness, and the mass set the characteristic frequency of the servo $\nu = 1/2\pi\sqrt{g_P/m}$. Because we have other oscillators in the antenna, for example, the dish with its 8.2 Hz characteristic frequency, we need to keep the frequency of the servo below that of all the other oscillators so that none of them will be driven in resonance. The lowest of these appears to be the dish. The combined inertia of the motor+screw and the dish is 110,000 kg m^2. Thus we select a proportional gain so that $\nu = 1/2\pi\sqrt{110000/g_P} < 8.2$ Hz. For example a proportional gain of 500 Nm/arcsec will result in a characteristic frequency of about 5 Hz. In principle, the damping term is then set so that the servo oscillator is about critically damped. Applied to the antenna, the result is shown in figure 3.

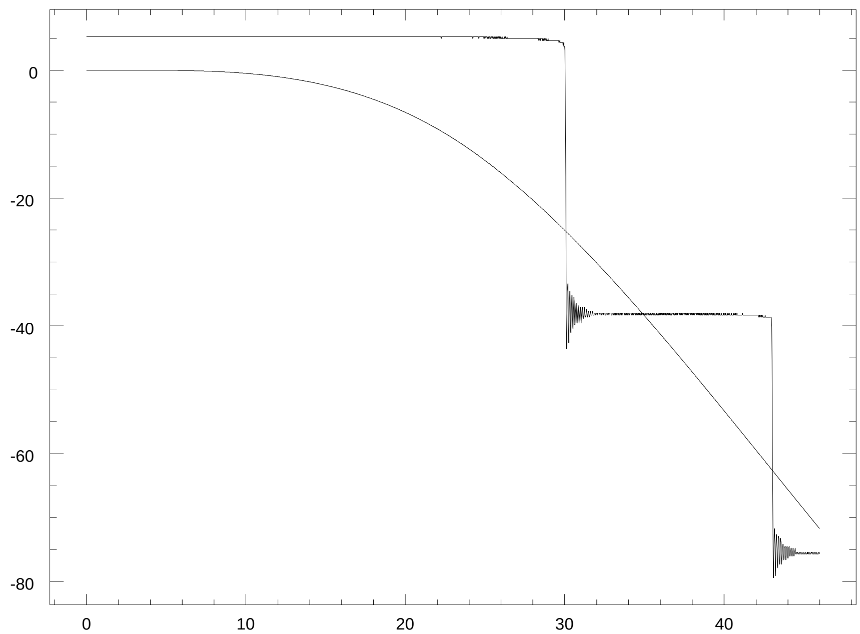

Figure 3. Elevation axis encoder counts in arc seconds versus time in seconds attempting to follow a smooth tracking command.

What is happening in this test is that stiction has locked the motor until the tracking error becomes large enough that there is sufficient commanded torque to break the stiction. For example, $30" \times 500$Nm/arcsec $= 15000$ Nm or about 50 Nm after dividing by a gear ratio of 300. So the tracking error must become about $30''$ before the servo is capable of making any correction. Once moving, the motor is then off and running. The commanded torque becomes smaller and smaller as the tracking error decreases. At some point depending on the momentum and the running friction, the motor+screw locks up. The dish then continues to oscillate around the locked motor for about 1/2 second. For this unsatisfactory system there is a simple way to estimate the tracking error; it is the stiction divided by the proportional gain. For this type of servo control, we cannot turn the proportional gain up any higher without driving the dish in resonance and the system unstable, so these errors are the best we can do. Figure 4 shows the model of the system which captures the essence of the problem.

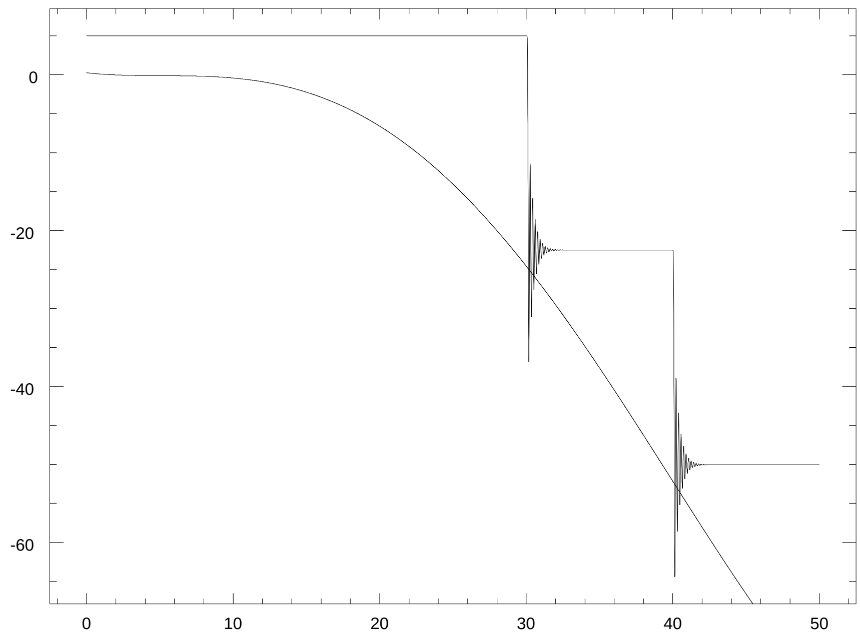

Figure 4. Modeled elevation axis encoder counts in arc seconds versus time in seconds attempting to follow a smooth tracking command. The modeled stiction is 13000 Nm, friction 3000 at an elevation angle of 77°.

If we run the feedback off the motor instead of the dish, then we have the advantage that the feedback loop does not have a resonance. In effect, we move the motor where we want and the dish comes along, oscillating to be sure, but the dish is not in the feedback loop, the resonance is not driven, and the oscillations will die down over time. We can of course monitor the position of the dish and make corrections on a time scale slow enough to avoid driving the dish in resonance. The disadvantage is that we do not know where the dish is pointing on a more rapid time scale, and the system will therefore inevitably be slower. Nonetheless we can turn the proportional gain way up in the hopes of reducing the steps. Figures 5 and 6 show the performance for this test. These measurements were made at 85° elevation (figure 5) where the dish is about as balanced as it ever gets and the stiction is the lowest and at 45° elevation (figure 6) where the stiction is about double. The proportional gain is set to 5000 Nm/arcsec. At a gain of 7000 Nm/arcsec, the current amplifier for the motor always shuts off due to excessive current. So for the present system, this performance is as good as we can get. While we still have a low resolution encoder, $\sim 1''$ on the motor, the performance appears to be limited by the stiction and the gain. (There is a reasonable speculation that the gain limit is related to resonance in the encoder, and stiffening the encoder, increasing its resolution, applying a low pass filter..., might allow a higher gain.) Figures 7 and 8 show the modeled behavior for these tests. For the lower elevation test we need to double the stiction and friction in the model because the stiction/friction is higher in the antenna at lower elevations.

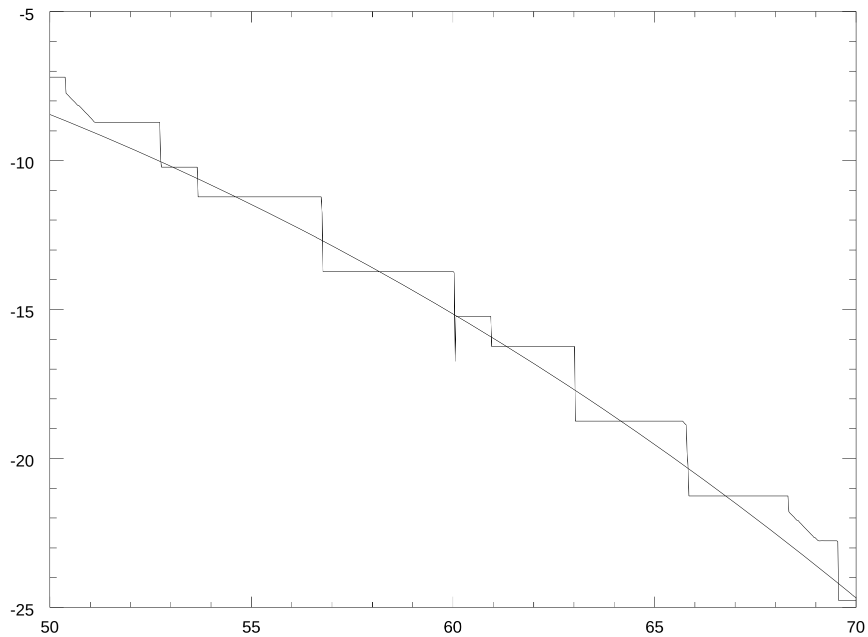

Figure 5. Measured motor encoder counts in arc seconds versus time in seconds. The proportional gain is 5000 Nm/arcsec and the elevation angle is 85°. The steps are about 3″.

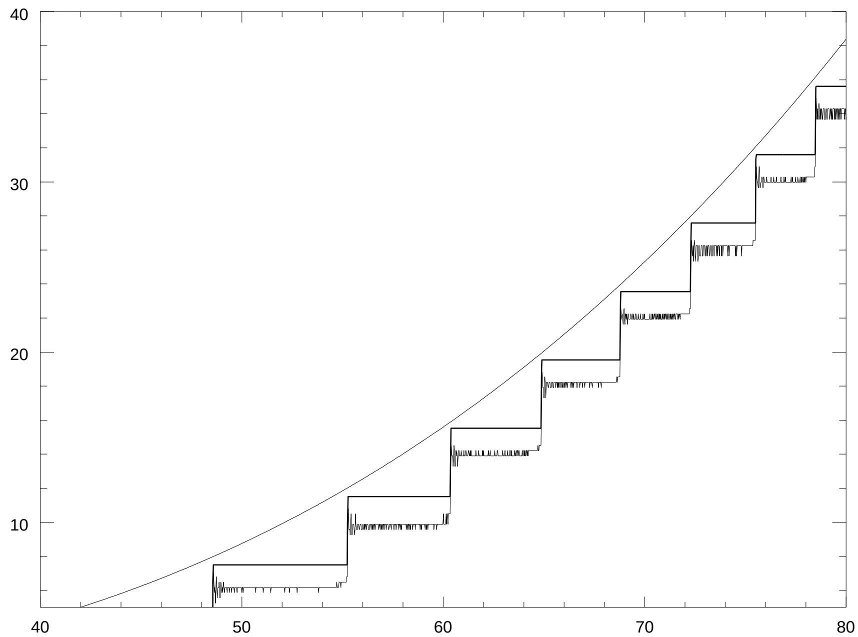

Figure 6. Measured motor and elevation encoder counts in arc seconds versus time in seconds. The line with the high frequency oscillations is the elevation axis and the line with simple steps is the motor. The smooth curve is the commanded track. The proportional gain is 5000 Nm/arcsec and the elevation angle is 45°. The steps are about $6''$

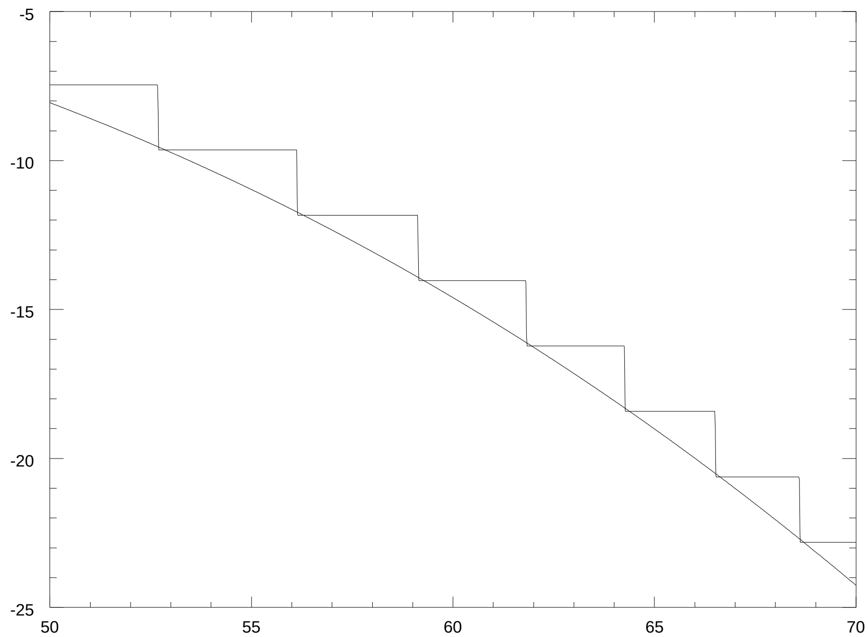

Figure 7. Modeled motor encoder counts in arc seconds versus time in seconds. The proportional gain is 5000 Nm/arcsec and the elevation angle is 85°. The stiction and friction are set to 50 ft lb and 15 ft lb. The steps are about 3″.

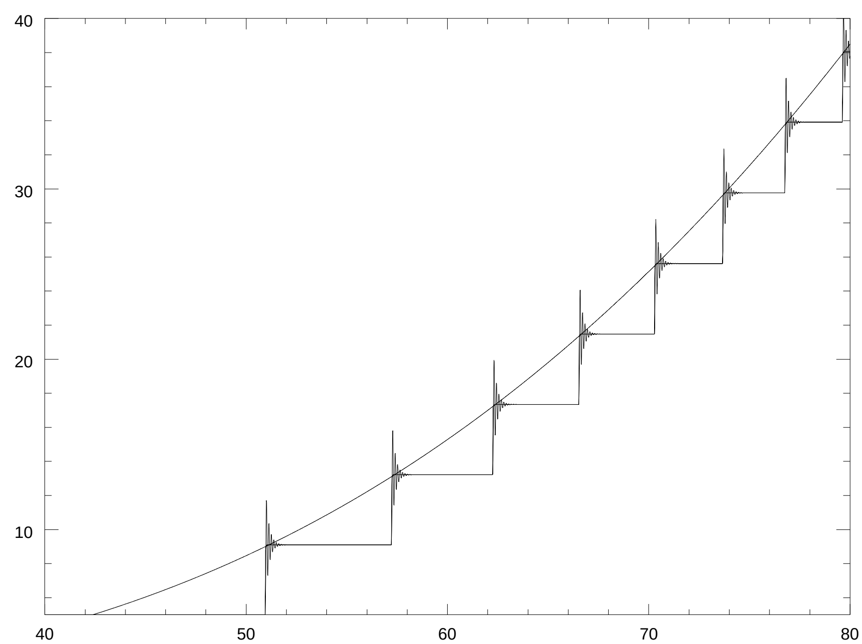

Figure 8. Modeled elevation axis encoder counts in arc seconds versus time in seconds. The proportional gain is 5000 Nm/arcsec and the elevation angle is 45°. The stiction and friction are set to 100 ft lb and 30 ft lb. The steps are about 6″.

Now we can ask the question, how much must the stiction/friction be reduced to meet the servo following error specification of 0.6″ rms. From the results of our experiments, the size of the steps in the tracking is proportional to the stiction. So to reduce the step size from 3″ to 1″ we need to reduce the stiction by a factor of 3. Figure 9 shows a modeled prediction for the same case as figure 5, elevation angle of 85°, proportional gain of 5000 Nm/arcsec, except the stiction and friction have been reduced in the model to 20 ft lb and 10 ft lb. This performance will technically meet the specification: the peak to peak error is 1.25″ and the rms error is 0.72″. The rms could be reduced by centering the steps about the track. Although this is within the specification, nobody would ever have designed a tracking servo to behave in this way. The servo is tracking at a rate of about 1″ per second. Every second of time, the antenna makes a 1″ jump, exciting the 8.2 Hz oscillation which then takes 1/2 second to damp down.

In setting the limit on the allowed stiction/friction, remember that there is a variation in the stiction/friction of at least 25%, a variation in the gear ratio, and the motor and amplifier are already running at about the maximum current draw. Allowing just for the variations in stiction of 25% implies that we should be thinking of a average level of stiction of about 15 ft lb.

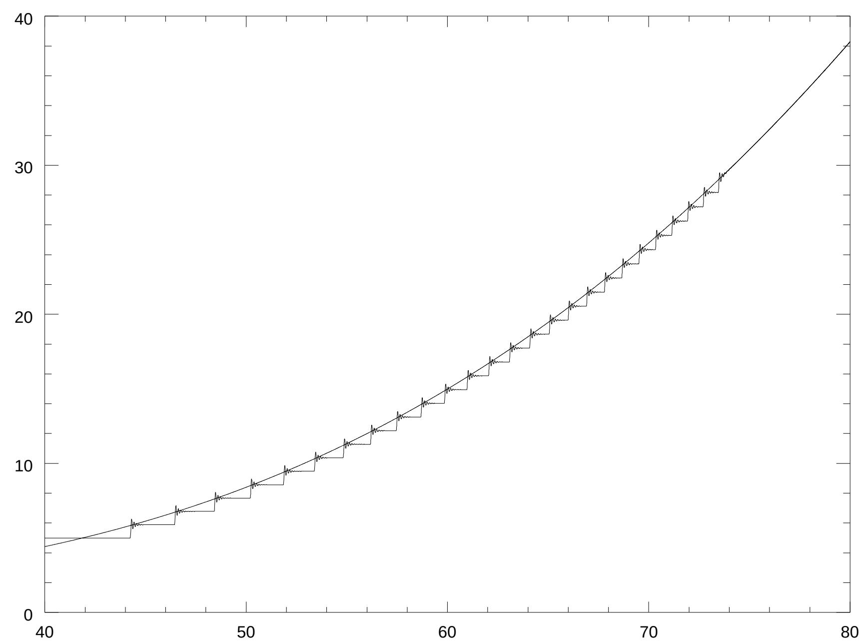

Figure 9. Modeled elevation axis encoder counts in arc seconds versus time in seconds. The proportional gain is 5000 Nm/arcsec and the elevation angle is 85°. The stiction and friction are set to 20 ft lb and 10 ft lb.

Constant Velocity Control However, it is possible to design a servo system which can handle a higher level of stiction/friction than the current one and which uses many of the same components. One way to overcome the high stiction and keep the system tracking smoothly is to keep the motor turning at all times by a tight velocity feedback loop on the motor. The current digital encoders do not easily adapt from position to velocity feedback, nor apparently does the primitive and inflexible PMAC computer. So we have not yet been able to test such a system. Nonetheless, we can model it, and with a modest effort and some additional hardware, it could be built and tested. The generic problem with this type of servo control is that a very tight velocity loop will keep the motor turning but tend to resist changes in velocity as would be required to move the antenna from say source to calibrator. As a result the system is slow to respond. Figure 10 presents a simulation of a combined velocity and position loop, suggested by P. Cheimets. The stiction and friction are at the current minimum values, that is about 60 ft lb and 20 ft lb. There are four gains in this loop; each is set to one-third the gain which was found to fault the amplifier, so the motor and amplifier could probably handle this loop. We would need a higher resolution encoder on the motor, or even better, a tachometer to test this loop on the antenna.

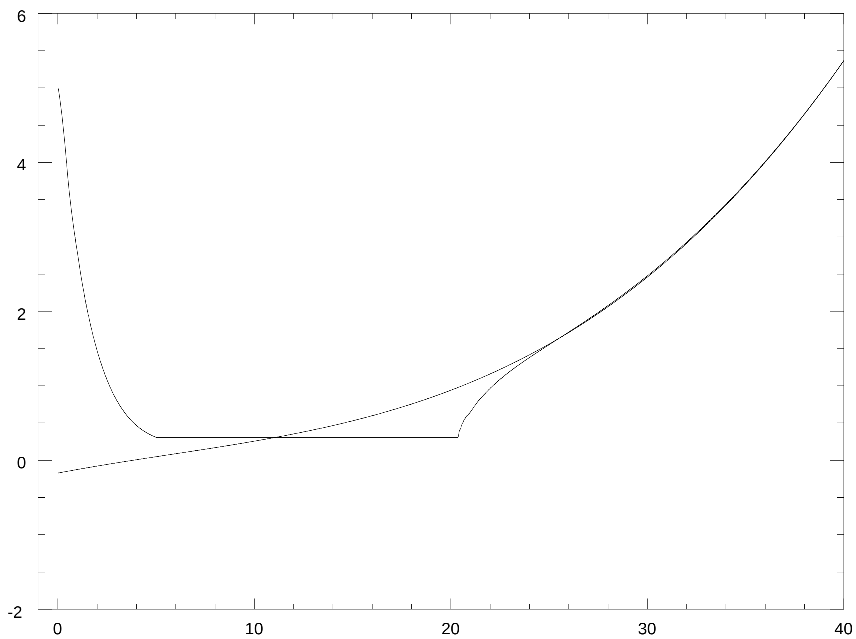

Figure 10. Modeled elevation axis encoder counts in arc seconds versus time in seconds for a combined velocity plus position loop. The elevation angle is 85°. The stiction and friction are set to 60 ft lb and 20 ft lb. The smoother curve is the commanded track. The antenna starts from a 5 arc second offset.

What is happening in this test, is that the antenna is asked to move onto a track from an initial offset 5 arc seconds away. The antenna requires 2 seconds of time to get within 1 arc second of the track at which point stiction locks the system. When the tracking error is large enough to command sufficient torque to break the stiction, the motor starts turning and thereafter tracks quite smoothly. From this simulation, one can see that the acquisition time depends on the gains and the friction, and that stiction is still a problem at the slowest speeds. This servo system does not meet the specifications for acquisition time: only 1 second is allowed to move from an offset of 5 arc *minutes*. Also the tracking error is too high at the slowest speeds. However, the system does track smoothly at higher speeds.

Another possibility, suggested by R. Wilson, is to set up a tight velocity loop with the commanded velocity proportional to the tracking error.

$$\dot{x} = k(x - x_{ref})$$

The solution of this differential equation is,

$$x_{ref} - x = e^{-kt}$$

showing that the antenna moves asymptotically onto the requested track with a time constant equal to the proportionality constant, k. At the moment we do not have a model which would allow us to determine the shortest time constant that our amplifier and motor could handle, nor do we have the hardware to test it. A rule of thumb consideration yields 0.5 seconds.

Conclusions The performance of the elevation tracking system is currently so poor as to render the array nearly useless at the higher frequency observations. The root cause is excessive stiction/friction relative to the type of servo system and current degree of stiffness of the antenna. If the stiction/friction can be reduced to about 15 ft lb or better at all elevation angles above 20°, then the current servo system can probably be made to work. Alternatively, the current servo system must be fixed to handle a greater gain. One possibility is to redesign the servo system as a velocity loop.

Acknowledgements All data used in these experiments were taken by Nimesh Patel and Robert Wilson.