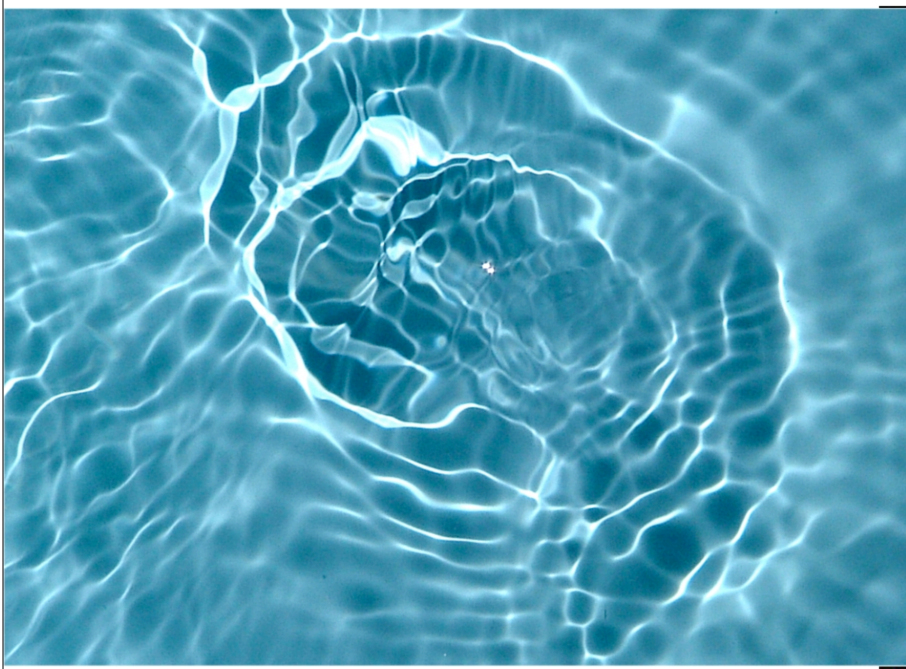
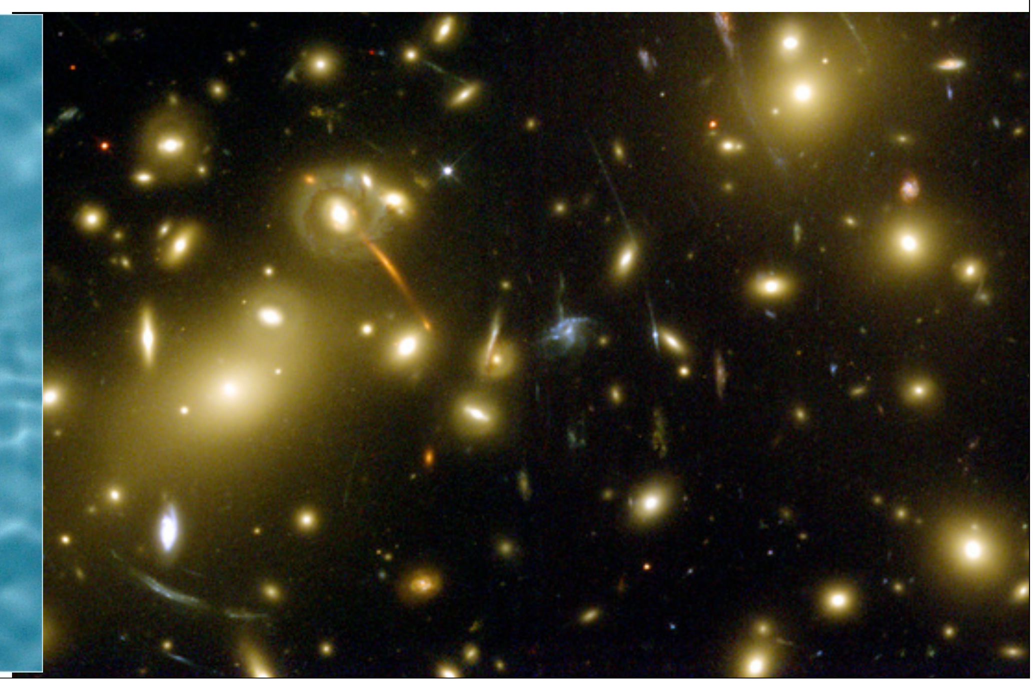
Gravitational lensing

Jonathan Pritchard





References

In preparing this lecture I have shamelessly stolen material from the following sources:

- Narayan & Bartelmann [astro-ph/9606001] - Jerusalem Winter School notes
- Schneider - Extragalactic Astronomy and Cosmology (2006)
- Schneider, Ehlers, & Falco - Gravitational Lenses (1999)
- Carroll - Spacetime and Geometry (2004)

My office is P-243, feel free to stop by if you have questions

Slides will be available from my website by the end of the day
http://www.cfa.harvard.edu/~jpritchard/teaching.htm

Outline

- Brief history

  1. Predictions back in the '30s
  2. Discovery of 0957+561
  3. Rediscovery of cluster arcs
  4. Nearest lens (2237+0305)

- Simple theory

  1. Deflection Angle
  2. Lens Equation
  3. Magnification

- Microlensing

  1. Probabilities/Timescales
  2. MACHO results

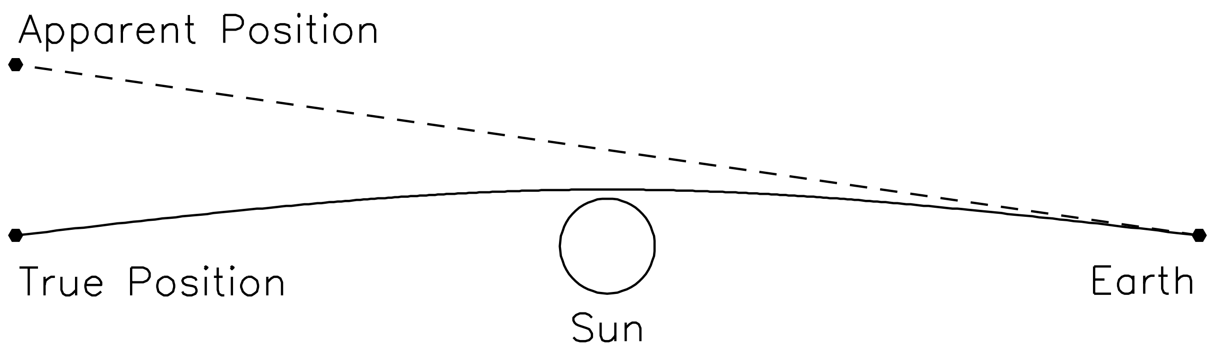
- Galaxy lenses

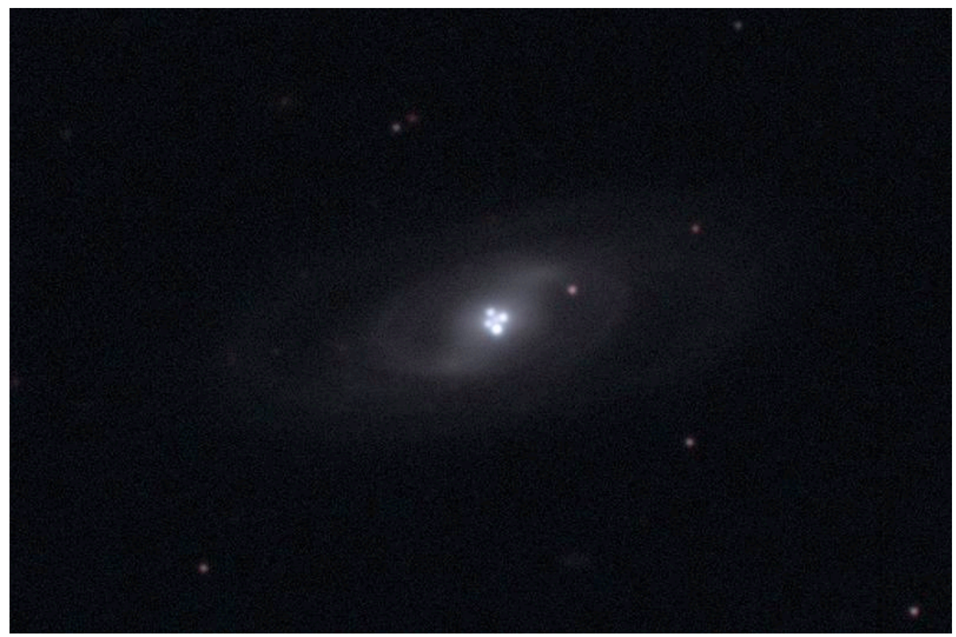
  1. Point masses
  2. Isothermal spheres
  3. Mass determinations
  4. Distances and H0

- Cluster & LSS lenses

  1. Mass determinations
  2. Weak lensing+cluster searches
  3. Cosmic sheer

History of Lensing





Newtonian and GR predictions

Gravitational lensing first proposed by Soldner (1801) in context of Newtonian theory. He found a deflection angle

$$\alpha = \frac{2GM}{v^2 r}$$

For sun gives 0.85"

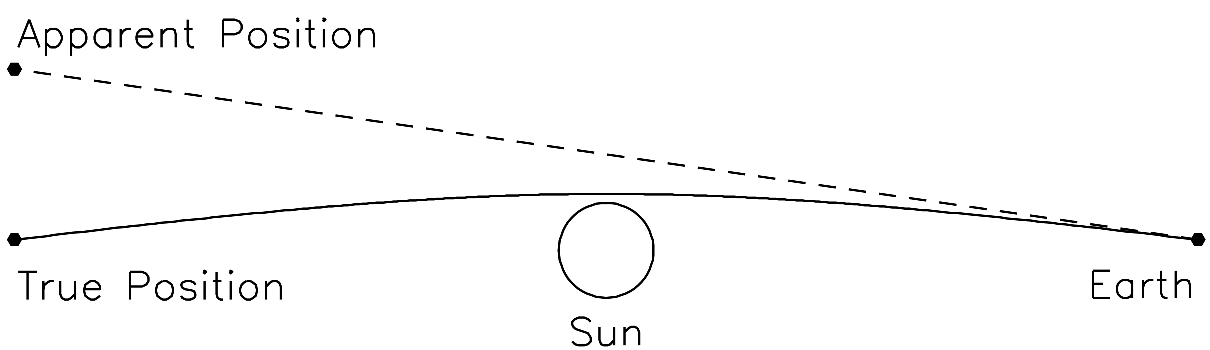
Einstein derived same result in 1911 using Equivalence principle & Euclidean metric

Freundlich's efforts to measure this during an Eclipse in the Crimea were foiled by the outbreak of WWI and his arrest by the Russians...

In 1915 with general relativity, Einstein derived the new result

$$\alpha = \frac{4GM}{c^2 r}$$

For sun gives 1.7"



Eddington and the Eclipse

Using data taken during a solar eclipse in 1919, Eddington measured a value close to that of the GR prediction

Dyson, Eddington, & Davidson 1920

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 331

The result from declinations is about twice the weight of that from right ascensions, so that the mean result is

$$1''\cdot 98$$

with a probable error of about $\pm 0''\cdot 12$.

The Principe observations were generally interfered with by cloud. The unfavourable circumstances were perhaps partly compensated by the advantage of the extremely uniform temperature of the island. The deflection obtained was

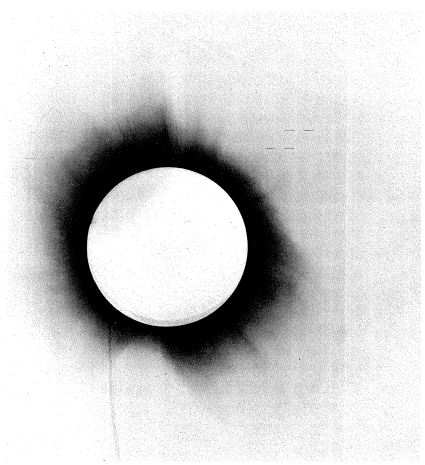
$$1''\cdot 61.$$

The probable error is about $\pm 0''\cdot 30$, so that the result has much less weight than the preceding.

Both of these point to the full deflection $1''\cdot 75$ of EINSTEIN's generalised relativity theory, the Sobral results definitely, and the Principe results perhaps with some uncertainty. There remain the Sobral astrographic plates which gave the deflection

$$0''\cdot 93$$

discordant by an amount much beyond the limits of its accidental error. For the reasons already described at length not much weight is attached to this determination.



332 SIR F. W. DYSON, PROF. A. S. EDDINGTON AND MR. C. DAVIDSON ON A

Thus the results of the expeditions to Sobral and Principe can leave little doubt that a deflection of light takes place in the neighbourhood of the sun and that it is of the amount demanded by EINSTEIN's generalised theory of relativity, as attributable to the sun's gravitational field. But the observation is of such interest that it will

(bizzarely if not for this then Eddington might well have been imprisoned for being a pacifist)

Zwicky's leap

- Although calculations of lensing by other stars were carried out the small angular separations of the images led to pessimism that they could be seen
- In 1937, Zwicky made the jump of suggesting that extragalactic nebulae (galaxies) would produce well separated images that could be observed
  - by applying the virial theorem to the Coma and Virgo clusters he was (correctly) using masses ~400 times larger than was then believed
- He pointed out that gravitational lensing would allow the study of objects at greater distances (via magnification), that many arcs should be visible, and the importance of magnification bias in magnitude limited samples.

Zwicky 1937

Nebulae as Gravitational Lenses

The discovery of images of nebulae which are formed through the gravitational fields of nearby nebulae would be of considerable interest for a number of reasons.

(1) It would furnish an additional test for the general theory of relativity.

(2) It would enable us to see nebulae at distances greater than those ordinarily reached by even the greatest telescopes. Any such *extension* of the known parts of the universe promises to throw very welcome new light on a number of cosmological problems.

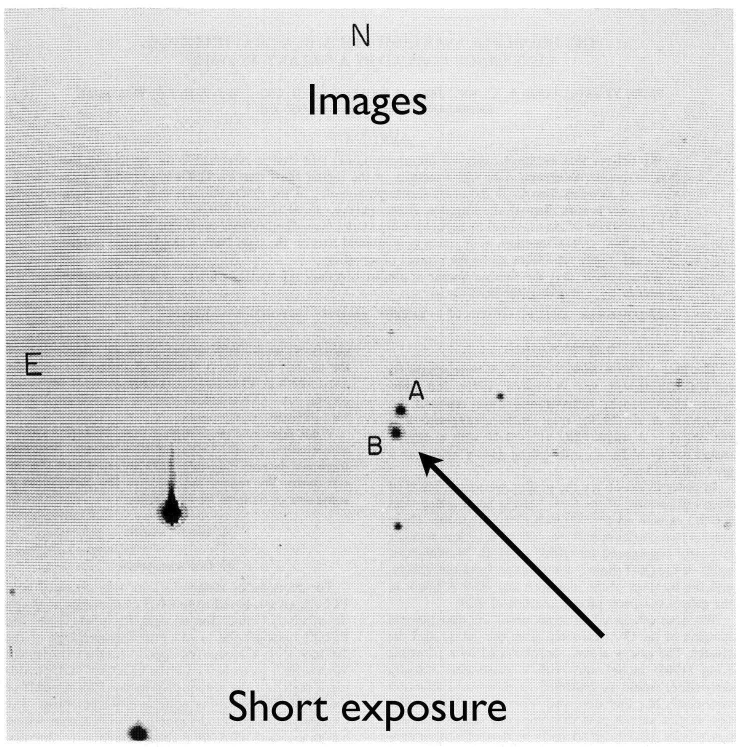
(3) The problem of determining nebular masses at present has arrived at a stalemate. The mass of an average nebula until recently was thought to be of the order of $M_N = 10^9 M_\odot$, where $M_\odot$ is the mass of the sun. This estimate is based on certain deductions drawn from data on the intrinsic brightness of nebulae as well as their spectrographic rotations. Some time ago, however, I showed[2] that a straightforward application of the virial theorem to the great cluster of nebulae in Coma leads to an average nebular mass four hundred times greater than the one mentioned, that is, $M_N' = 4 \times 10^{11} M_\odot$. This result has recently been verified by an investigation of the Virgo cluster.[3] Observations on the deflection of light around nebulae may provide the most direct determination of nebular masses and clear up the above-mentioned discrepancy.

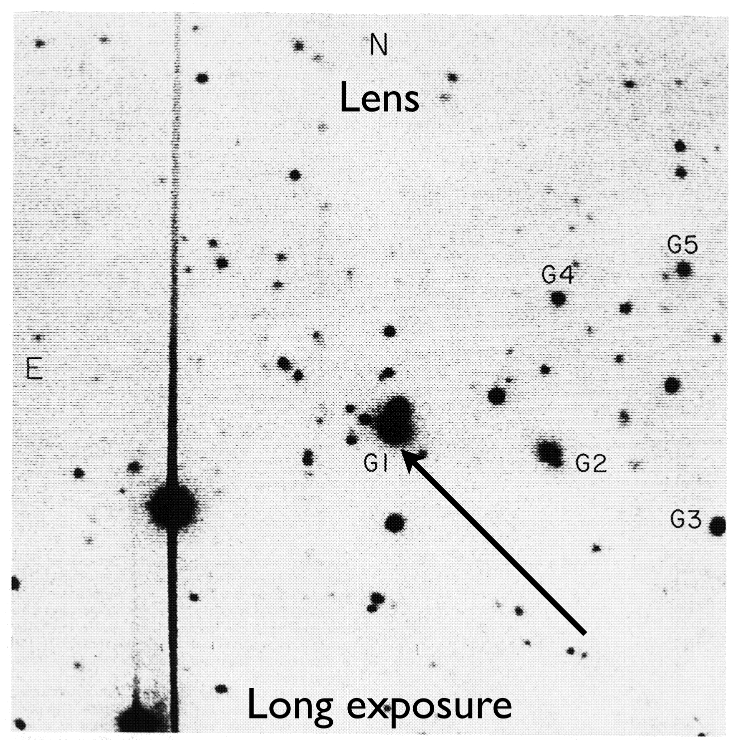
Discovery of 0957+561

- The first concrete example of a gravitational lens was reported in 1979 in the form of the quasar QSO 957+561 A,B found at z~1.4 (Walsh, Carswell & Weymann 1979). Two seen images separated by 6".

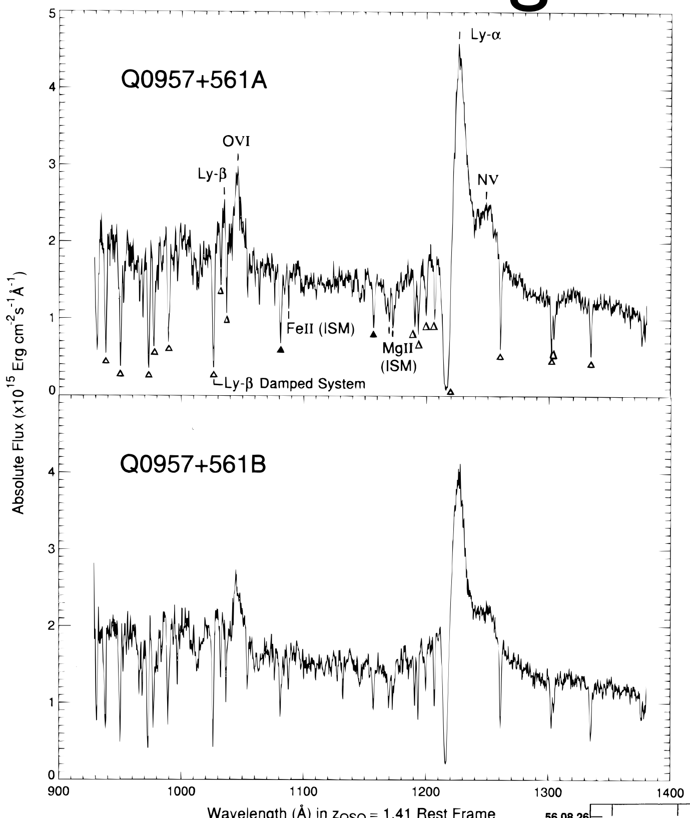
- Evidence that this is a lens comes from

    1. Lensing galaxy detected at z~0.36
    2. Similarity of the spectra of the two images
    3. Ratio of optical and radio fluxes are consistent between two images
    4. VLBI imaging showed detailed correspondence between small scale features





Images of QSO 0957+561



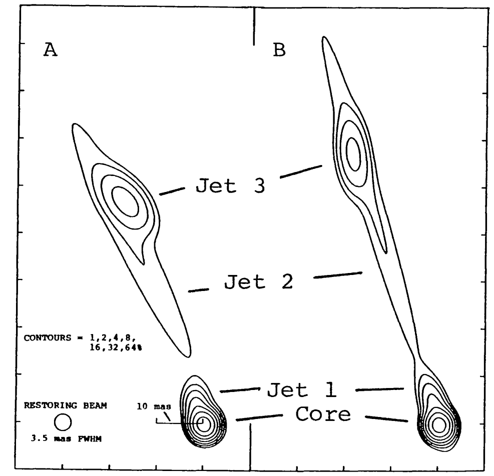
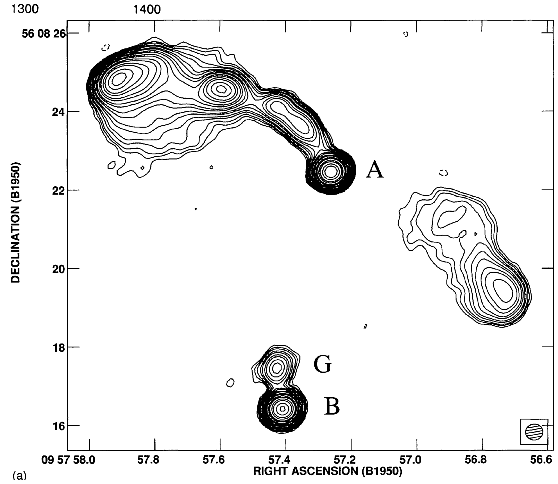
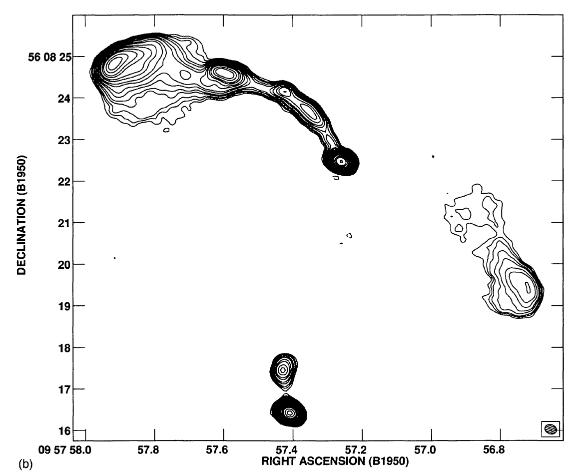


Figure 1 — The brightness distributions of 0957+561A and B. Four elliptical Gaussian components account for the data obtained from the 1981 λ13 cm VLBI observations for A and for B (Model I—see text). The correspondences in the number and in the properties of the components of the respective models, and the evidence for a change in parity from one image to the other, all support the hypothesis that A and B are images of a single object.

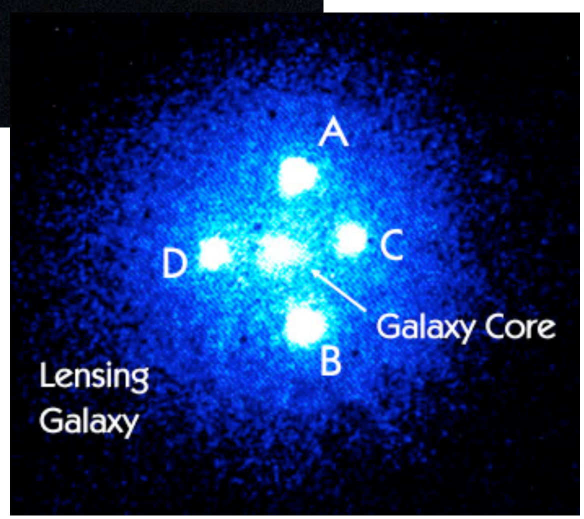


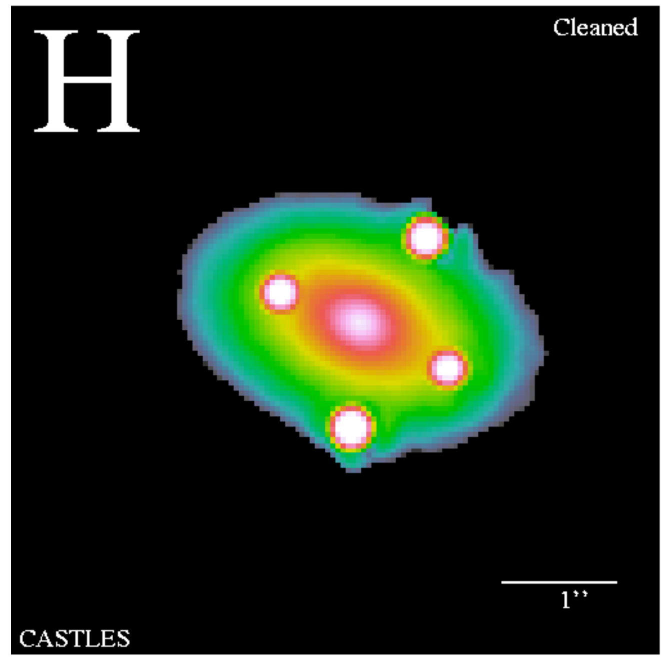


“Huchra’s Lens”

- Quadruply-imaged quasar Q2237+0305 “Einstein Cross” z=1.7 with image separation ~1.8” -> elliptical lens
- Lensing galaxy is ZW2237+030 “Huchra’s Lens” at z=0.04

Huchra+(1985)

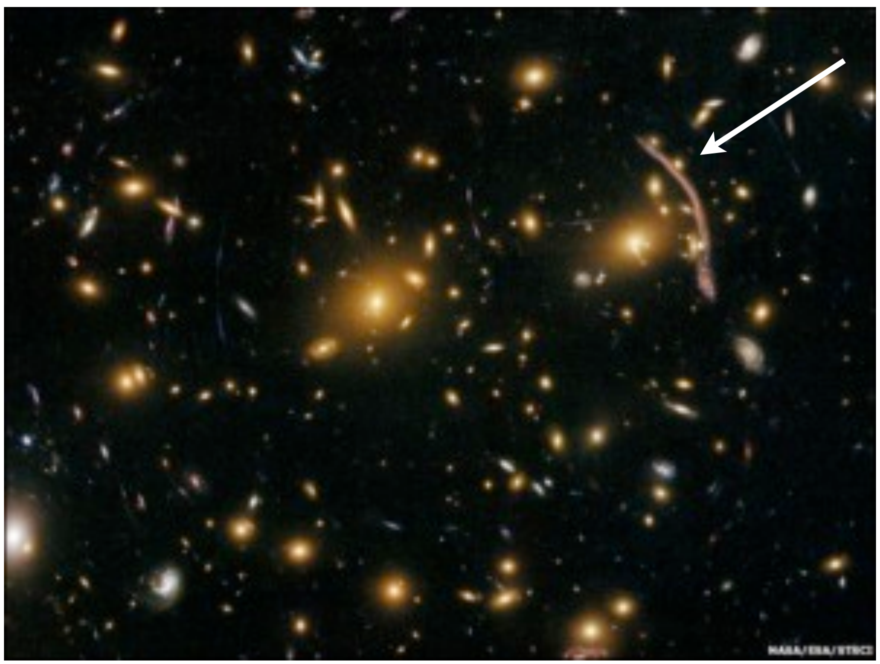




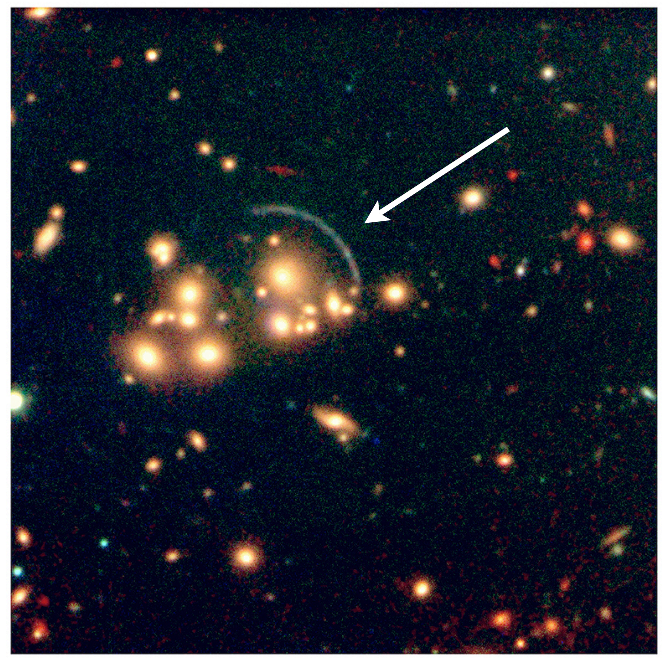
Nearby and isolated
-> key system for testing GR

Cluster Arcs

- In 1986, two groups discovered stretched arcs in clusters of galaxies at high redshift. “giant luminous arcs” - very thin in radial direction (unresolved)
- Light from arc confirmed to be from a much higher redshift source
- Confounded expectations based on pre-ROSAT X-ray observations that the surface mass density of clusters was too low to cause strong lensing
- Suddenly everyone found arcs in their old data...



Abel 370 - HST



IR Colour Composite of Galaxy Cluster CL2244-02 with Gravitational Arcs
(VLT UT1 + ISAAC)

ESO PR Photo 46d/98 (26 November 1998)

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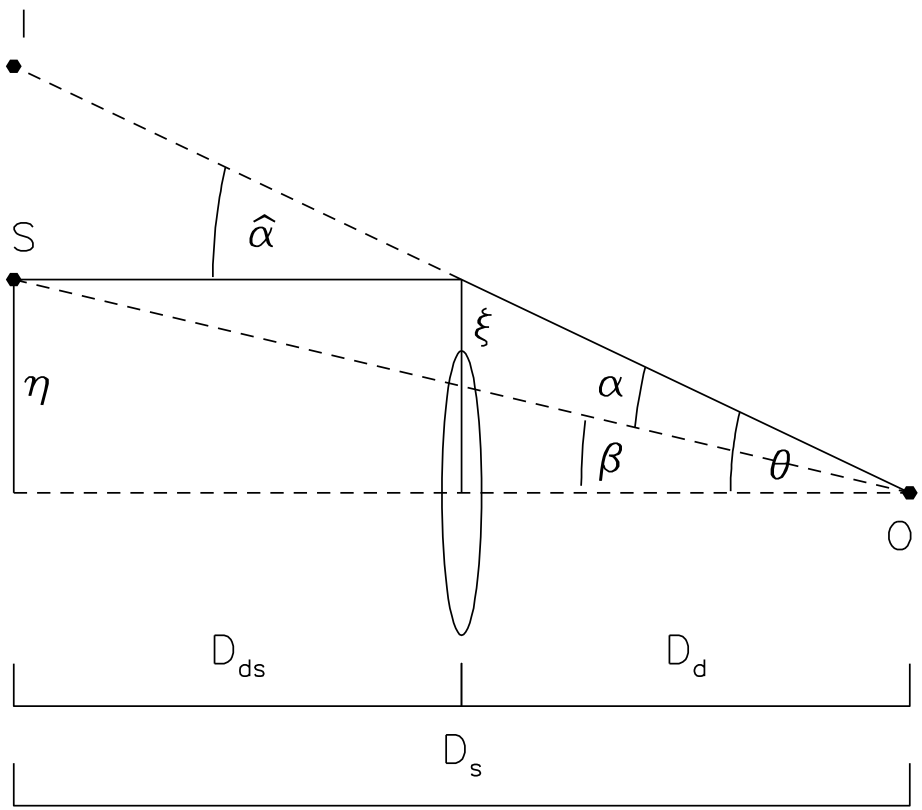
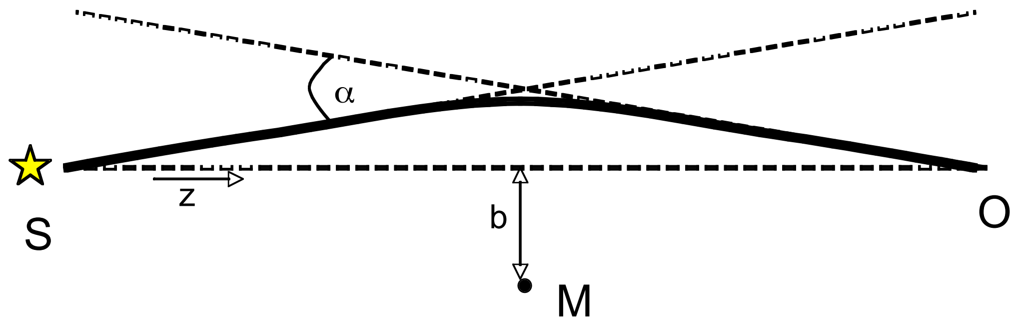
Basic Theory

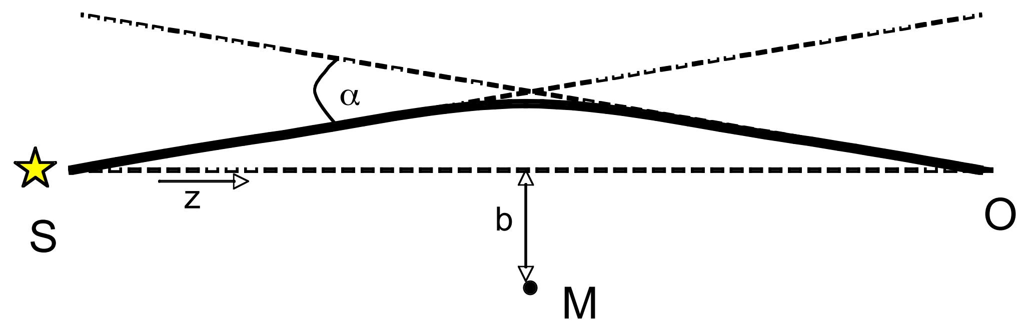


Gravitational Deflection



- Derive gravitational deflection angle α from GR - just sketch elements here see e.g. Carroll for a complete treatment
- Metric: $$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)(dx^2+dy^2+dz^2)$$
- Poisson Equation: $$\nabla^2\Phi = 4\pi G\rho$$
- Geodesic equation: $$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{\mathrm{d}x^\rho}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\sigma}{\mathrm{d}\lambda} = 0$$
- Null path condition: $$g_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\nu}{\mathrm{d}\lambda} = 0$$
- Solve for photon path assuming that deflection is small so can treat as a small perturbation and integrate along the undeflected path to obtain
- Deflection angle: $$\hat{\alpha} = 2\int \nabla_\perp \Phi \,\mathrm{d}s$$

Example: Point mass



Start here:

$$\hat{\alpha} = 2 \int \nabla_\perp \Phi \, \mathrm{d}s \qquad \nabla^2 \Phi = 4\pi G \rho$$

Poisson eq. gives potential:

$$\Phi(r) = -\frac{GM}{r} = -\frac{GM}{(b^2 + z^2)^{1/2}} \qquad \nabla_\perp \Phi(r) = \frac{\partial \Phi}{\partial b} = -\frac{GMb}{(b^2 + z^2)^{3/2}}$$

Deflection angle follows:

$$\begin{aligned} \hat{\alpha} &= 2 \int \nabla_\perp \Phi(r) \\ &= 2 \int_{-\infty}^{\infty} \mathrm{d}z \, \frac{GMb}{(b^2 + z^2)^{3/2}} \\ &= \frac{2GM}{b} \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{(1 + x^2)^{3/2}} \\ &= \frac{4GM}{b} \end{aligned}$$

$$\boxed{\hat{\alpha} = \frac{4GM}{b}}$$

Thin screen approximation

Most of the deflection occurs near to the lens (within z~b). Since the distances to the source and the observed are much greater than this we're motivated to make the thin screen approximation and treat all deflection as occurring in the lens plane

For a general mass distribution we get:

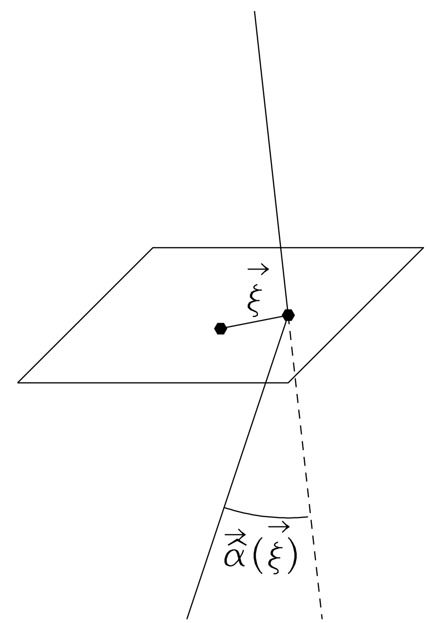
Projected surface density:

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z)\, dz$$

Then since weak deflections add linearly

Deflection angle:

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2}\, d^2\xi'$$



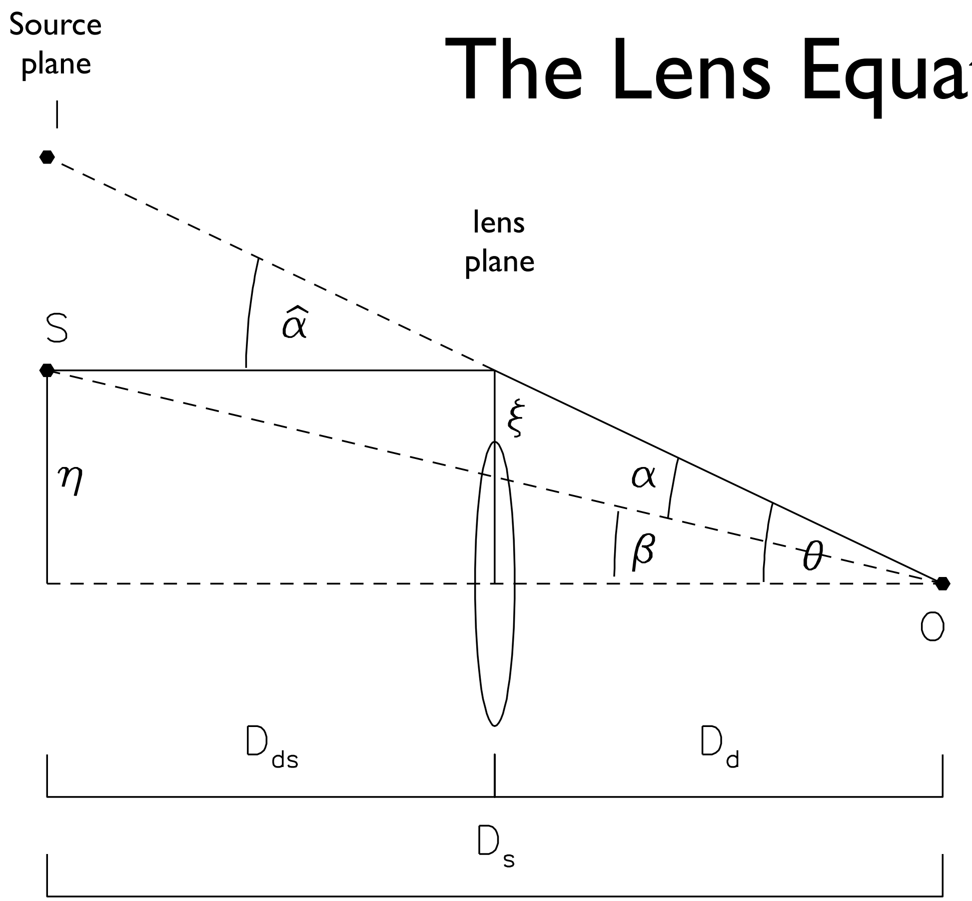
In the case of circular symmetry these look more familiar

$$\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2 \xi}$$

$$M(\xi) = 2\pi \int_0^{\xi} \Sigma(\xi')\xi'\, d\xi'$$

Now consider what this looks like to an observer...

The Lens Equation



$$\boldsymbol{\xi} = D_d \boldsymbol{\theta}$$

$$D_{ds} \hat{\boldsymbol{\alpha}} = D_s \boldsymbol{\alpha}$$

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{ds}}{D_s} \hat{\boldsymbol{\alpha}}(D_d \boldsymbol{\theta})$$

Small angles so: $\text{separation} = \text{angle} \times \text{distance}$

True in flat space, but is also how we define angular diameter distance in cosmology so holds more generally

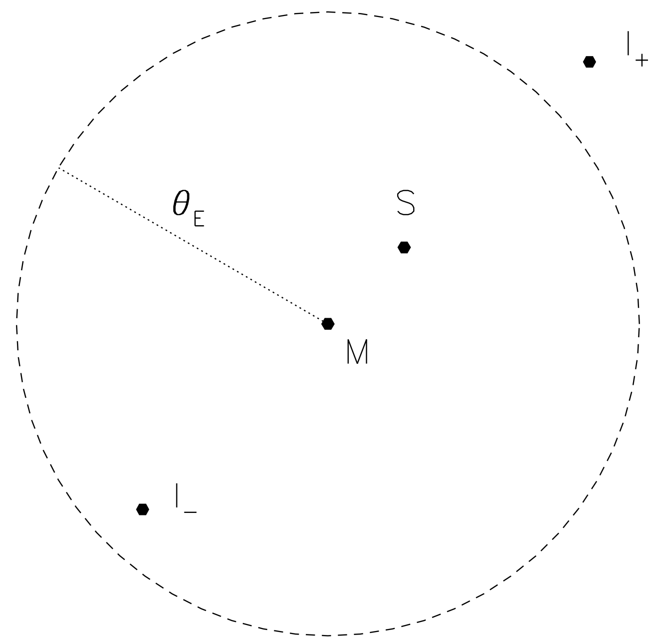
If space is not flat then $D_{\rm ds} \neq D_{\rm s} - D_{\rm d}$. and in principle lensing allows one of the few model independent tests of curvature in cosmology

Example: Point mass lens

$$\hat{\alpha} = \frac{4GM}{c^2|\xi|}$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{LS}}{D_S}\hat{\boldsymbol{\alpha}}(\boldsymbol{\theta})$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}\frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|^2}$$



Einstein Radius $\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}}$

Lens equation $\boldsymbol{\beta} = \boldsymbol{\theta} - \theta_E^2 \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|^2}$

$\beta > \theta_E$ source weakly lensed one weakly distorted image

$\beta < \theta_E$ source strongly lensed and multiple images

Two solutions $\theta_{\pm} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_{\mathrm{E}}^2}\right)$

Symmetry forces source, lens and images to lie on same line

Characteristic scales

Characteristic angles

$$\theta_{\rm E} = (0.9\,{\rm mas}) \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D}{10\,{\rm kpc}}\right)^{-1/2} ,$$

Lens: galactic star; source: LMC star

$$\theta_{\rm E} = (0\overset{\prime\prime}{.}9) \left(\frac{M}{10^{11}\,M_\odot}\right)^{1/2} \left(\frac{D}{\rm Gpc}\right)^{-1/2} .$$

Lens: galaxy; source: quasar

$$D = \frac{D_{\rm d} D_{\rm ds}}{D_{\rm s}} .$$

Critical density

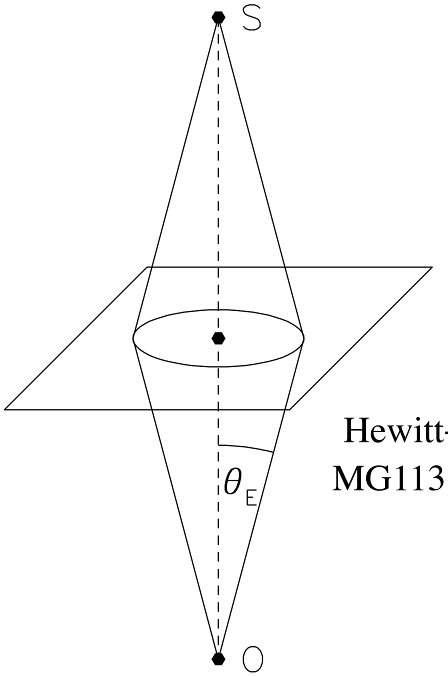
$$\Sigma_{\rm cr} = \frac{c^2}{4\pi G} \frac{D_{\rm s}}{D_{\rm d} D_{\rm ds}} = 0.35\,{\rm g\,cm^{-2}} \left(\frac{D}{1\,{\rm Gpc}}\right)^{-1}$$

Extended object needs $\Sigma > \Sigma_{cr}$ somewhere for strong lensing to occur

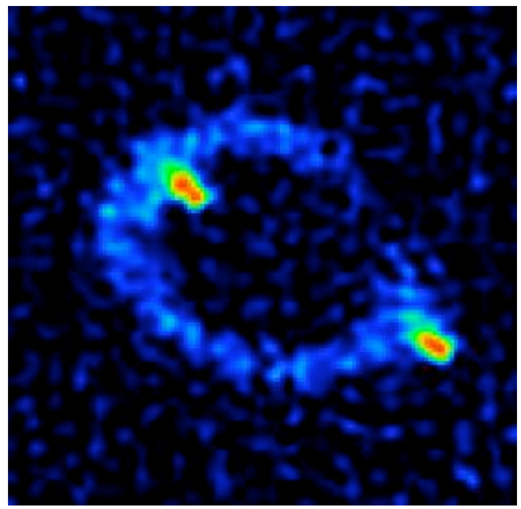
Einstein Ring

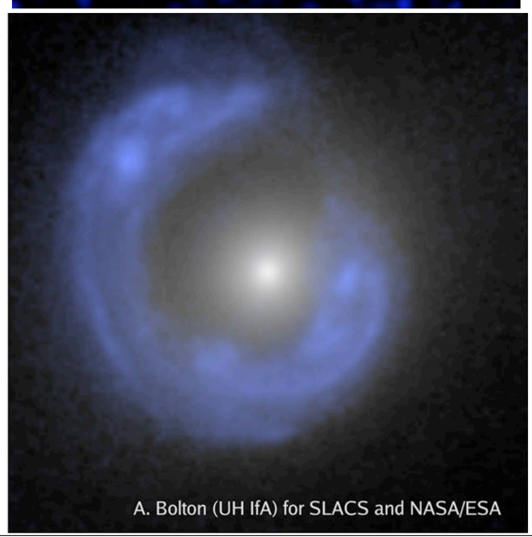
When lens, source, and observer lie on the same line get Einstein ring

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}}$$



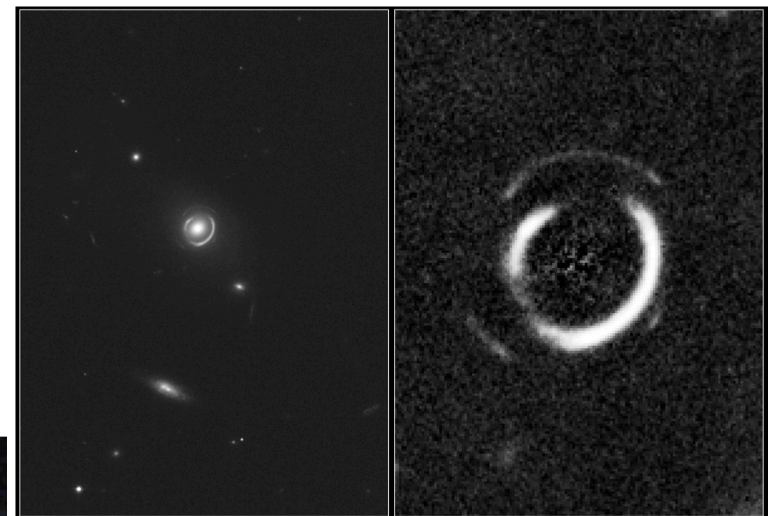
Hewitt+ 1987
MG1131+0456





SDSSJ1430

Double Einstein Ring - v. rare



Gavassi+ 2008

Einstein radius depends on lens mass AND geometry

Magnification

Gravitational lensing preserves surface brightness, but changes the apparent solid angle of the source => magnification

$$\text{magnification} = \frac{\text{image area}}{\text{source area}}$$

Magnification calculated using the lens equation as

$$\mu = \left|\det\left(\frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}}\right)\right|^{-1} \equiv \left|\det\left(\frac{\partial \beta_i}{\partial \theta_j}\right)\right|^{-1}$$

If circularly symmetric $\mu = \frac{\theta}{\beta}\frac{d\theta}{d\beta}$

Example: point mass

Images $\theta_\pm = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_{\rm E}^2}\right)$

$$u = \beta\theta_{\rm E}^{-1}.$$

magnification $\mu_\pm = \left[1 - \left(\frac{\theta_{\rm E}}{\theta_\pm}\right)^4\right]^{-1} = \frac{u^2+2}{2u\sqrt{u^2+4}} \pm \frac{1}{2}$

+ image always magnified
- image can go either way

total magnification $\mu = |\mu_+| + |\mu_-| = \frac{u^2+2}{u\sqrt{u^2+4}}$

Source on Einstein ring $\beta = \theta_{\rm E},\ u = 1$ $\mu = 1.17 + 0.17 = 1.34$

Shapiro time delay

Passage through potential also leads to time delay

$$\Delta t = -\int \Phi \, \mathrm{d}s$$

Shapiro 1964

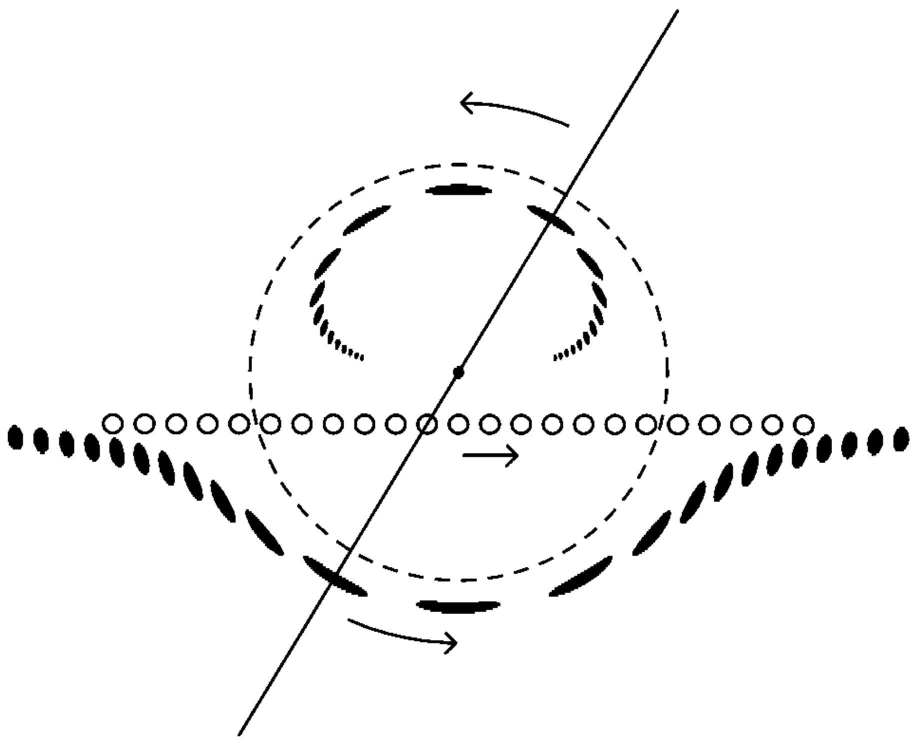
Complementary information to image locations
c.f. deflection angle only probes gradient of potential

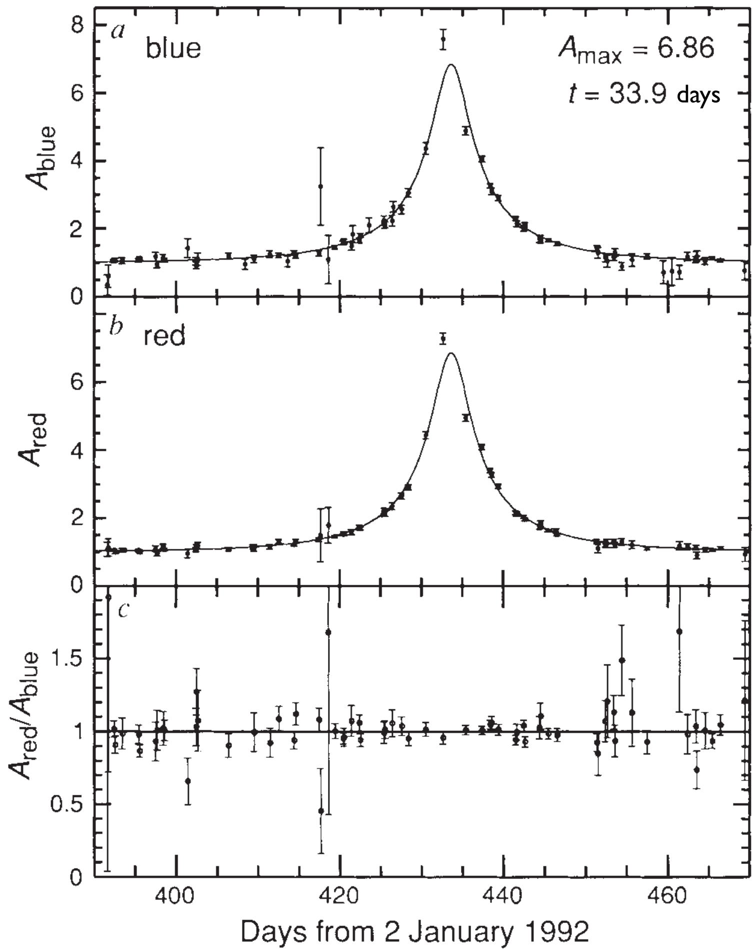
$$\hat{\alpha} = 2 \int \nabla_{\perp} \Phi \, \mathrm{d}s$$

Total time delay is the sum of the extra path length from the deflection and the gravitational time delay

$$\begin{aligned} t(\vec{\theta}) &= \frac{(1+z_{\mathrm{d}})}{c} \frac{D_{\mathrm{d}} D_{\mathrm{s}}}{D_{\mathrm{ds}}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right] \\ &= t_{\mathrm{geom}} + t_{\mathrm{grav}} \; . \end{aligned}$$

Microlensing





Microlensing

Einstein radius for a solar mass lens in the galaxy and a source located in the LMC

$$\theta_{\mathrm{E}} = (0.9\,\mathrm{mas}) \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D}{10\,\mathrm{kpc}}\right)^{-1/2}$$

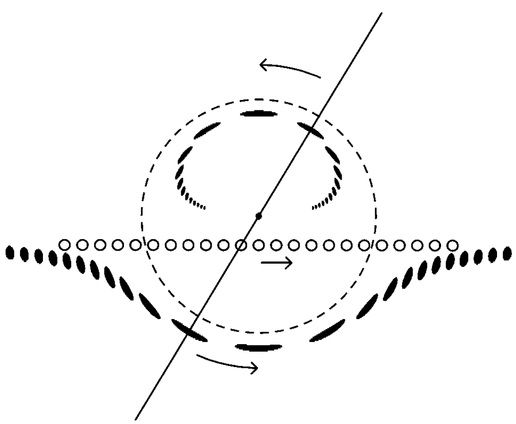
Too small to resolve individual images with optical telescope, but can see the effect of magnification

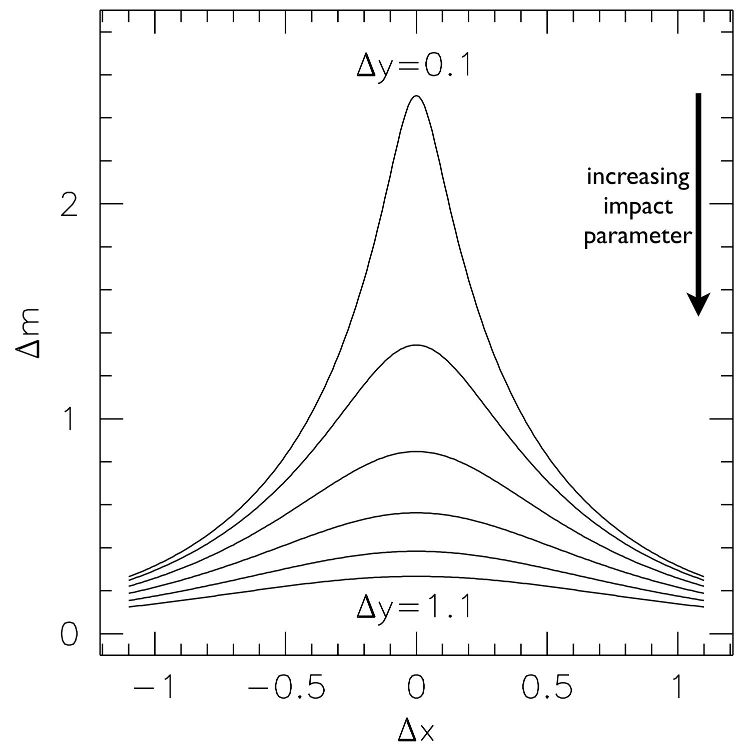
For sources that pass inside the lens Einstein radius

$$\mu \geq 1.34 \longrightarrow \mathrm{mag} \geq 0.32$$

Readily observable change in source magnitude

Light curve gives impact parameter and time scale





Time scales

Typical time scales range from hours to months

$$t_0 = \frac{D_{\rm d}\theta_{\rm E}}{v} = 0.214\,{\rm yr}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D_{\rm d}}{10\,{\rm kpc}}\right)^{1/2}\left(\frac{D_{\rm ds}}{D_{\rm s}}\right)^{1/2}\left(\frac{200\,{\rm km\,s^{-1}}}{v}\right)$$

Measured time scale therefore constrains mass of object, but is degenerate with distance to lens/source and velocity of lens

Degeneracy can be broken if velocity is changing - e.g. from parallax due to Earth's acceleration

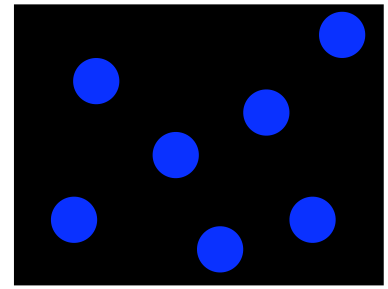
Can use this to search for dark massive objects in our galaxy
e.g. massive dark matter clumps = MACHOs
faint stars

Probability of microlensing

Optical depth= Probability at any instant of time that a given star is within θ_E of a lens

$$\tau = \frac{1}{\delta\omega} \int \underbrace{dV}_{\text{volume element}} \overbrace{n(D_\mathrm{d})}^{\text{number density of lenses}} \underbrace{\pi\theta_\mathrm{E}^2}_{\text{area of lens}}$$

$$dV = \delta\omega \, D_\mathrm{d}^2 \, dD_\mathrm{d}$$



$$\tau = \int_0^{D_\mathrm{s}} \frac{4\pi G\rho}{c^2} \frac{D_\mathrm{d} D_\mathrm{ds}}{D_\mathrm{s}} \, dD_\mathrm{d} = \frac{4\pi G}{c^2} D_\mathrm{s}^2 \int_0^1 \rho(x) \, x(1-x) \, dx$$

$$x \equiv D_\mathrm{d} D_\mathrm{s}^{-1}$$

$$D_\mathrm{ds} = D_\mathrm{s} - D_\mathrm{d}.$$

$$\tau = \frac{2\pi}{3} \frac{G\rho}{c^2} D_\mathrm{s}^2$$

Depends on mass density of lenses not on their mass.
Need times from light curve to get mass

Observed

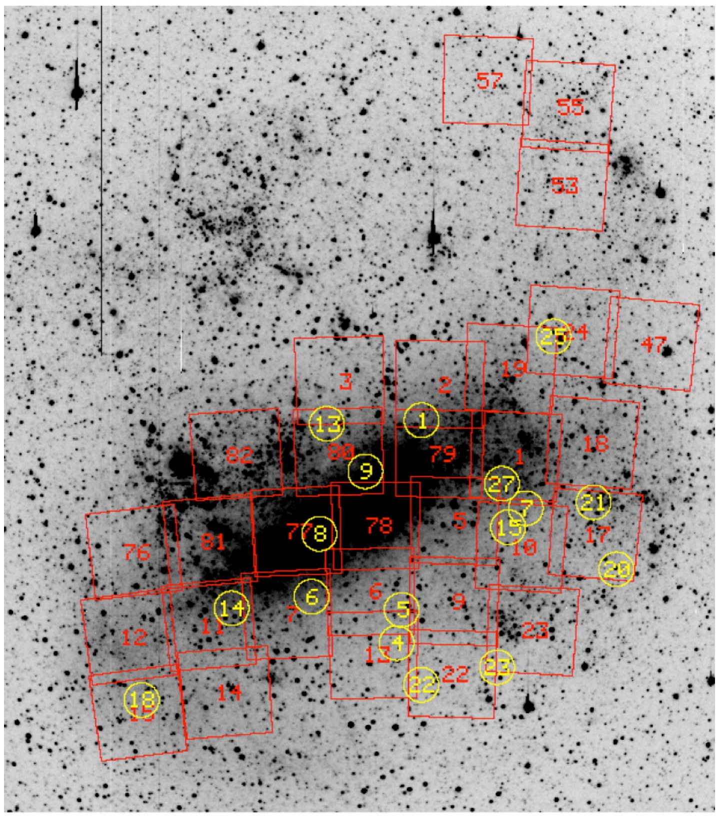
$$\tau(\mathrm{LMC}) \sim 10^{-7}$$

$$\tau(\mathrm{Gal.Bulge.}) \sim 10^{-6}$$

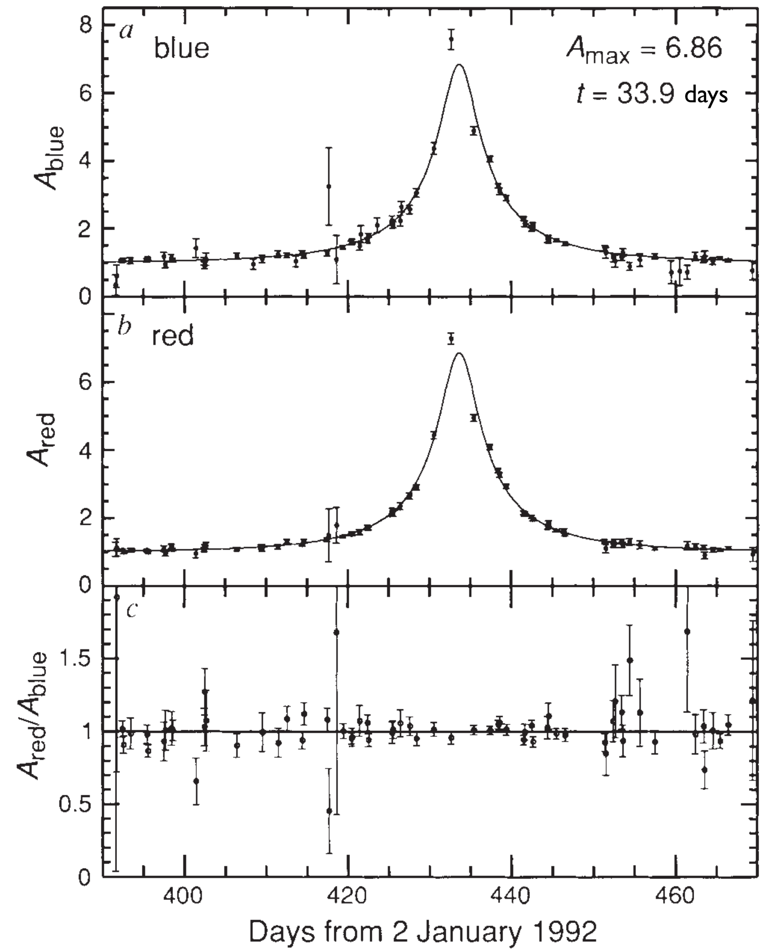
Low value means need to monitor millions of stars to see significant number of events

Microlensing requirements

17 microlensing events towards LMC
after ~5.7 years watching 11.9 million stars



Same lightcurve seen in two wavelength bands - used to exclude variable stars



Alcock et al (1993,2000)
MACHO collaboration

$t \sim 100\sqrt{M_{\rm macho}/M_\odot}$ days

t=33.9 days → M~0.12 Msol

Microlensing towards the bulge

- What is the distribution of stars in the galaxy?
- Looking for microlensing events towards galactic bulge useful calibration exercise for surveys since should definitely see something...
  1000s of events now been seen by OGLE, MACHO, MOA, EROS
- Prediction from known disc stars c. 1991 was

  $$\tau \sim 5 \times 10^{-7}$$ Paczyński (1991)

- Rises if lensing events between bulge stars is taken into account

  $$\tau \sim 8.5 \times 10^{-7}$$ Kiraga & Paczynski 1994

- Measured optical depth is considerably higher...

  $$\tau = 3.23^{+0.52}_{-0.50} \times 10^{-6}$$ Alcock et al. 2000b MACHO

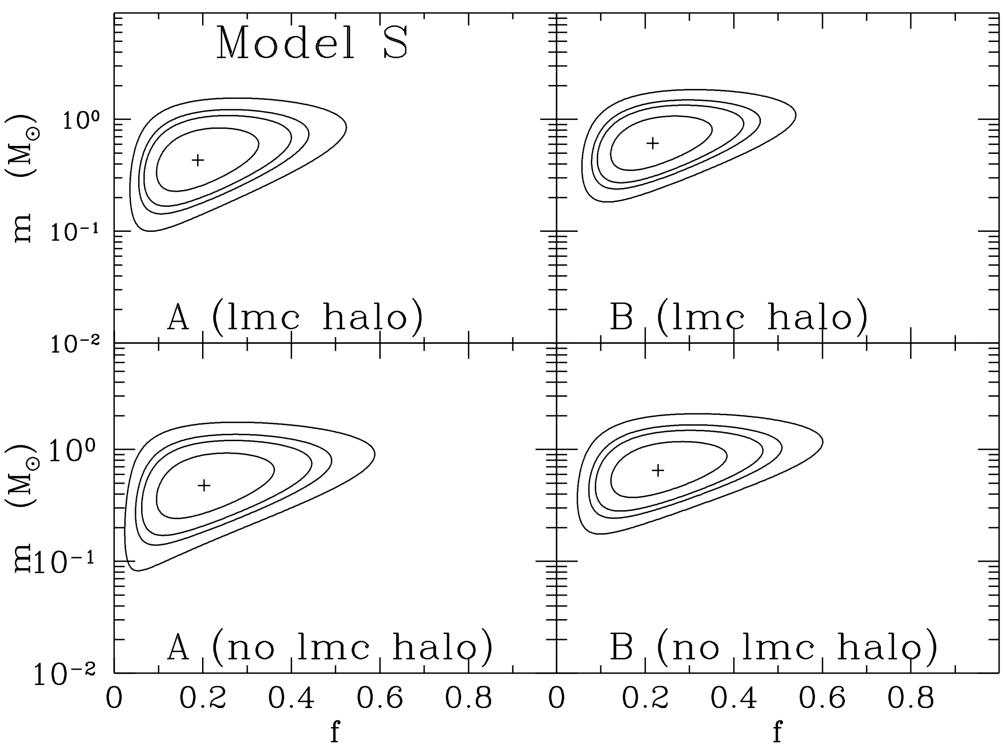
  $$\tau = 3.36^{+1.11}_{-0.81} \times 10^{-6}$$ Sumi+ 2003 MOA

- Accepted explanation is that this implies the existence of a bar in our galaxy aligned in the direction of the galactic center

Observations: MACHO survey

17 microlensing events towards LMC
after ~5.7 years watching 11.9 million stars → $\tau_2^{400} = 1.2^{+0.4}_{-0.3} \times 10^{-7}$

Larger than expected if no MACHOs...



What is the dark matter?
MACHO=Massive Compact Halo Object
WIMP = Weakly Interacting Massive Particle

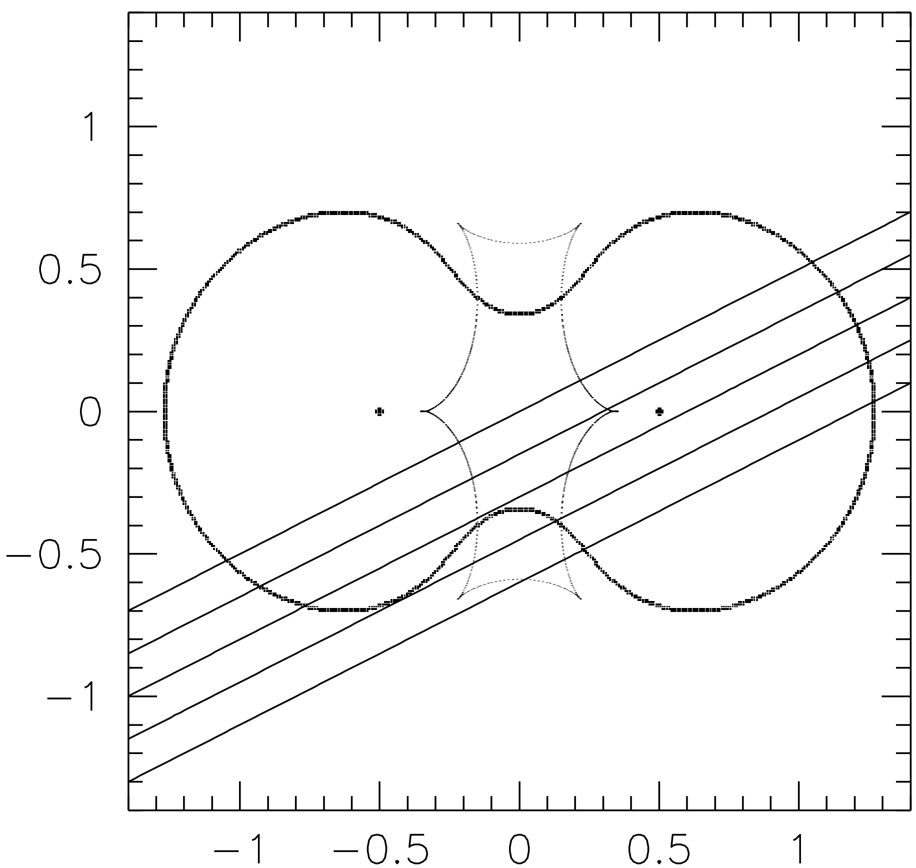
MACHO constraint is wierd. Suggests ~0.2 galaxy mass in ~0.5 Msol objects. Hard to explain...

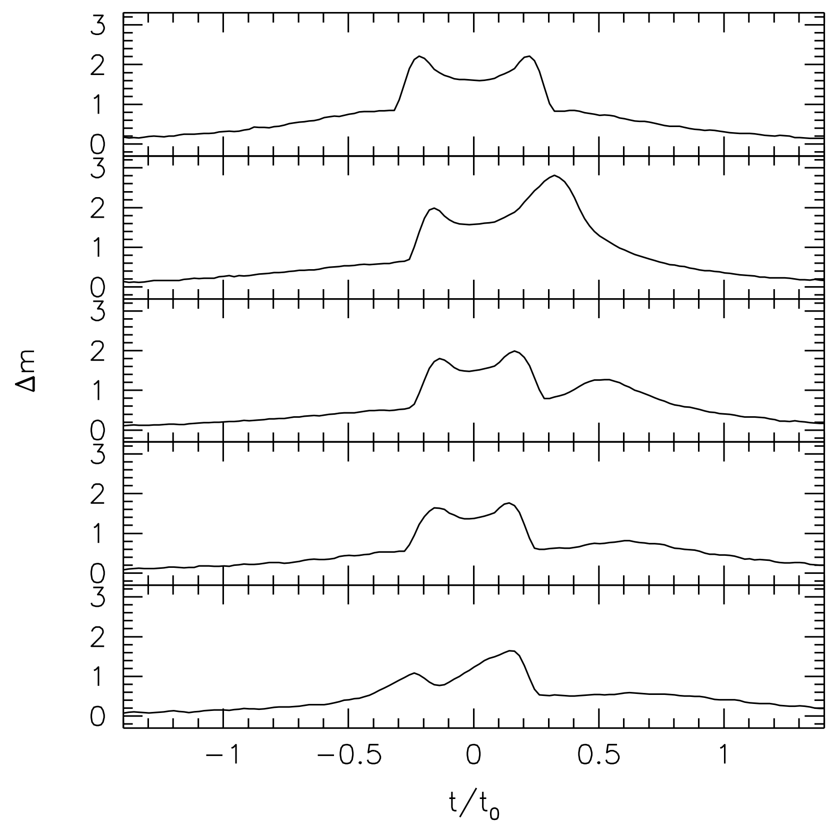
Too light to be white dwarves, neutron stars.

Alcock et al. 2000
(MACHO collaboration)

Binary lensing

About 10% of microlensing events are expected to be due to binary lenses

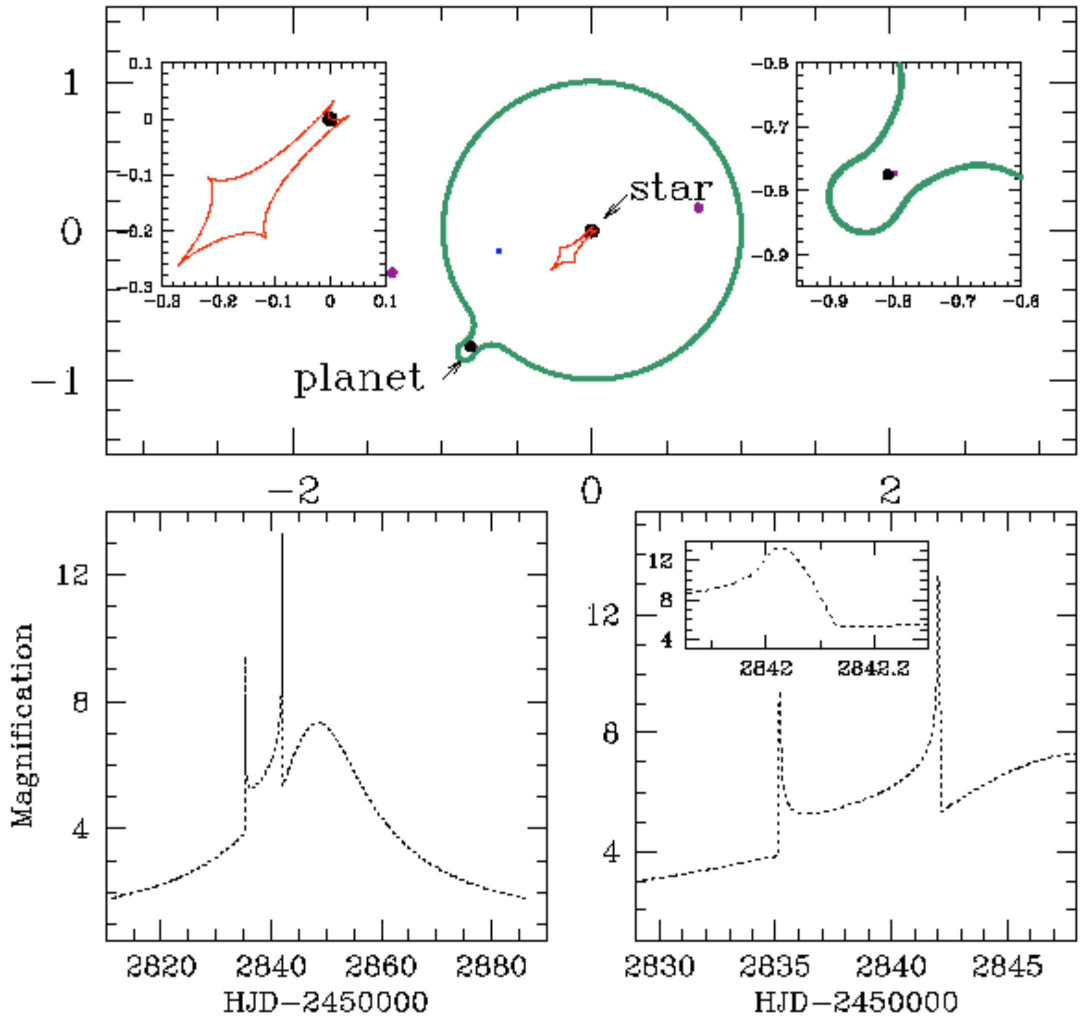




Double peaked light curves

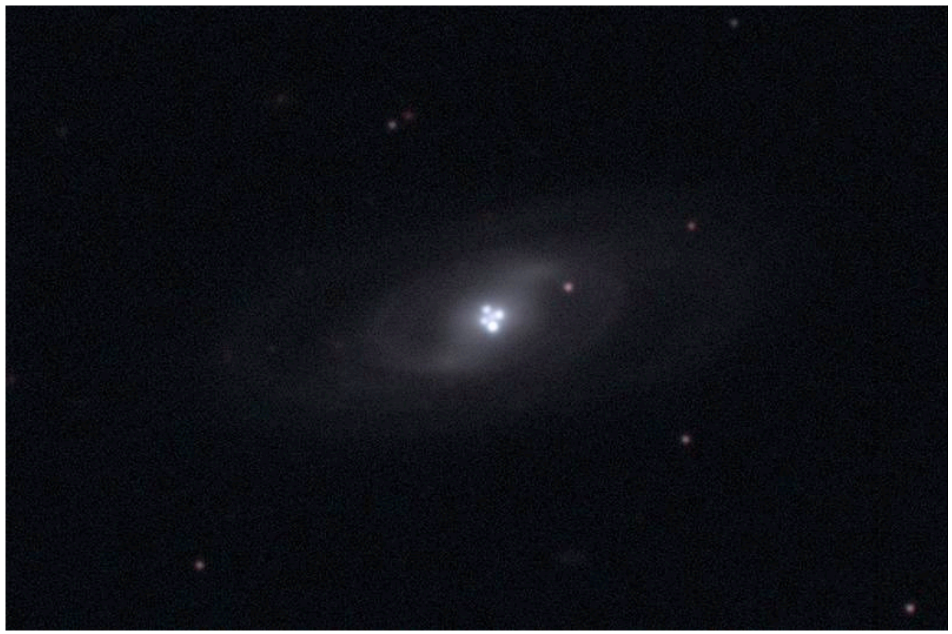
Planetary microlensing

OGLE 2003-BLG-235 - first planet detected via microlensing



Mstar~0.36 Msol   Mplanet~3.0 Mjupiter   separation~3AU   distance~5kpc

Galaxy lensing



Singular Isothermal Sphere

About 70 known gravitational lens systems with a galaxy as the lens

Galaxy lenses require that we account for the distributed nature of the mass.

Simple model assumes that stars and other mass behaves like particles of ideal gas

Ideal gas
$$p = \frac{\rho\, kT}{m} \qquad m\sigma_v^2 = kT$$

Hydrostatic equilibrium
$$\frac{p'}{\rho} = -\frac{GM(r)}{r^2}\;, \quad M'(r) = 4\pi\, r^2\, \rho\;,$$

Density profile
$$\rho(r) = \frac{\sigma_v^2}{2\pi G}\,\frac{1}{r^2}$$

(total mass divergent unless truncated at some radius)

Flat rotation curve
$$v_{\rm rot}^2(r) = \frac{G\,M(r)}{r} = 2\,\sigma_v^2 = \text{constant}\;.$$

Surface density
$$\Sigma(\xi) = \frac{\sigma_v^2}{2G}\,\frac{1}{\xi}$$

Constant deflection angle
$$\boxed{\hat{\alpha} = 4\pi\,\frac{\sigma_v^2}{c^2}}$$

SIS and lensing equation

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

$$\alpha = \hat{\alpha}\frac{D_{ds}}{D_s} = 4\pi\frac{\sigma_v^2}{c^2}\frac{D_{ds}}{D_s} = \theta_E$$

For strong lensing, get two images as for point mass

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \theta_E\frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|} \longrightarrow \theta_\pm = \beta \pm \theta_\mathrm{E}$$

Magnification can be very large for sources aligned with line to lens - Einstein ring again

$$\mu_\pm = \frac{\theta_\pm}{\beta} = 1 \pm \frac{\theta_\mathrm{E}}{\beta} = \left(1 \mp \frac{\theta_\mathrm{E}}{\theta_\pm}\right)^{-1}$$

Separation of the images is typically a few arc seconds for galaxy lenses

$$\theta_E = 1.6" \left(\frac{\sigma_v}{200\,\mathrm{km\,s^{-1}}}\right)^2 \left(\frac{D_{ds}}{D_s}\right)$$

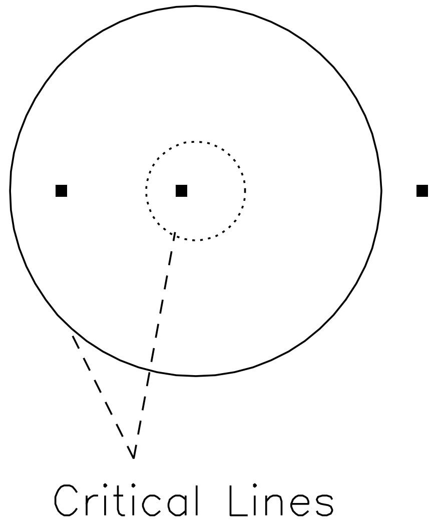
Although a good simple model in general the core of a galaxy would not be singular...

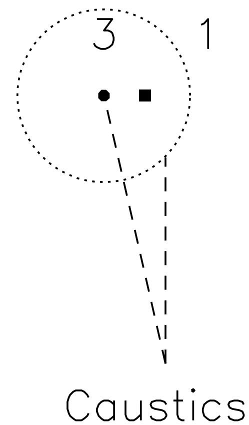
Caustics and Critical lines

lens plane

Source plane

Point source

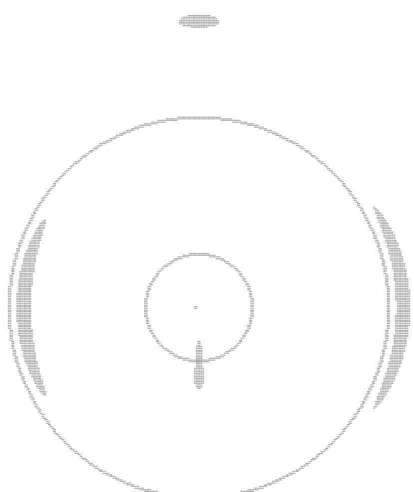


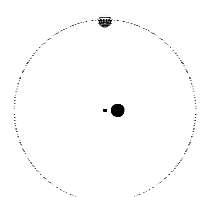


Only get central image if mass dist. is non-singular

point & circle

Extended source

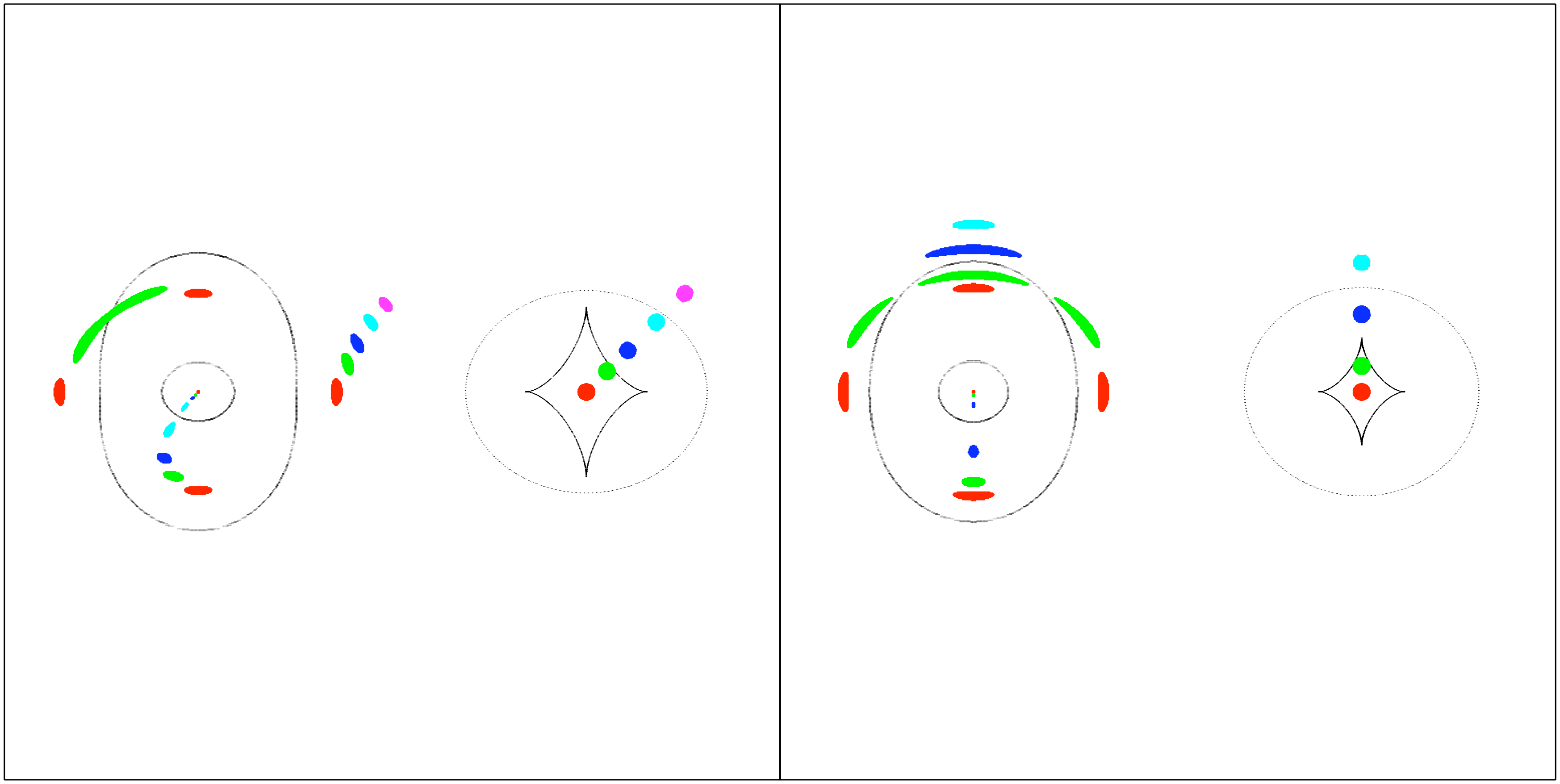


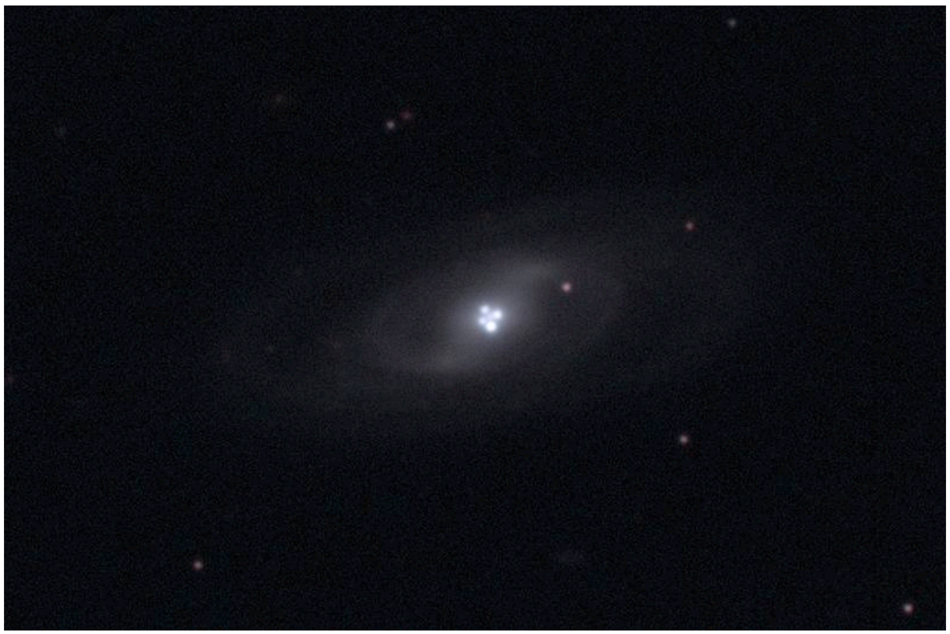


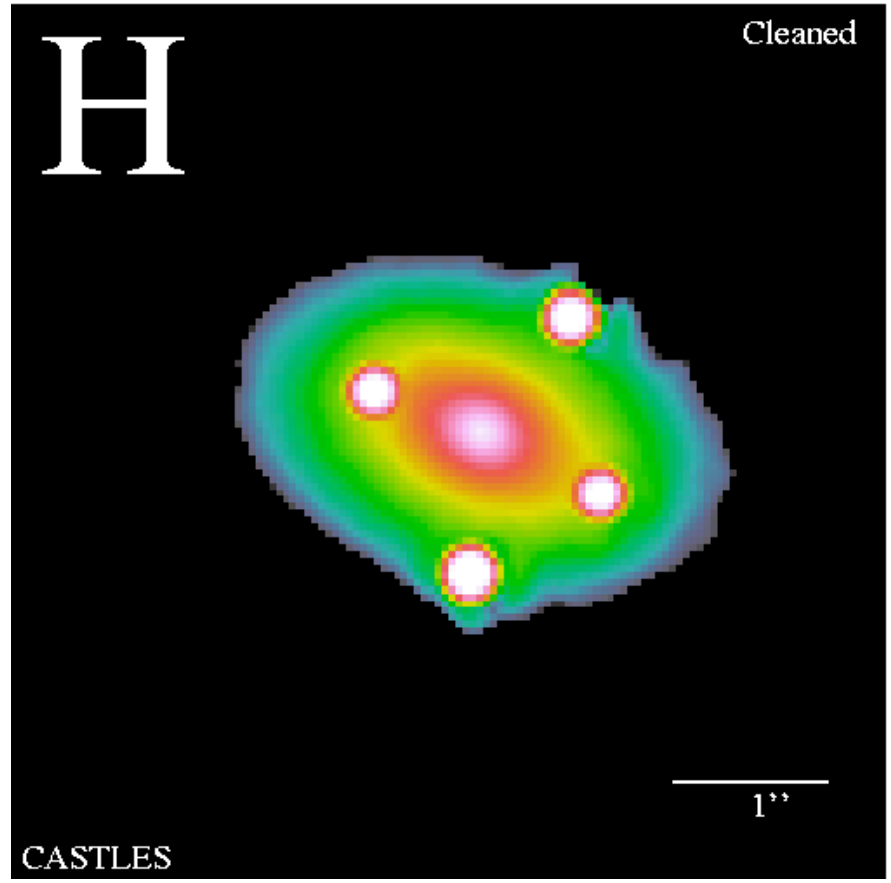
radial critical curves (inner)
tangential critical curves (outer)

Elliptical lenses

Elliptical lenses (or circular lens+external sheer) lead to new lensing configurations
Point caustic expands into diamond shape
Images no longer constrained to lie on a line
Source behind lens can lead to five images (one de-magnified)
On source crossing caustic images merge and disappear







Remember this?

Mass determinations

- As we saw earlier, when we have multiple images they tend to hover around the Einstein radius. We can use this to estimate the mass of the galaxy since the mean surface density inside the Einstein radius = critical mean density

$$M(\theta_E) = \pi(D_L\theta_E)^2\Sigma_{cr}$$

- This is a fairly simple estimate and in reality one would use the observed images to constrain a more detailed model of the lens e.g. as an isothermal ellipsoid (this is helped a lot if the source is extended since then more than four point images can be used to constrain the lens model e.g. Suyu+ (2009))

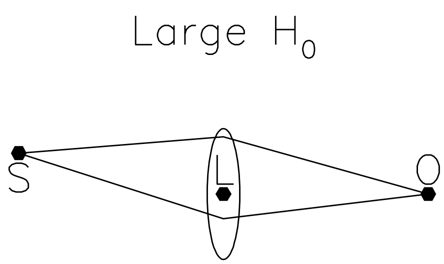
- Such mass constraints can be accurate to a few percent

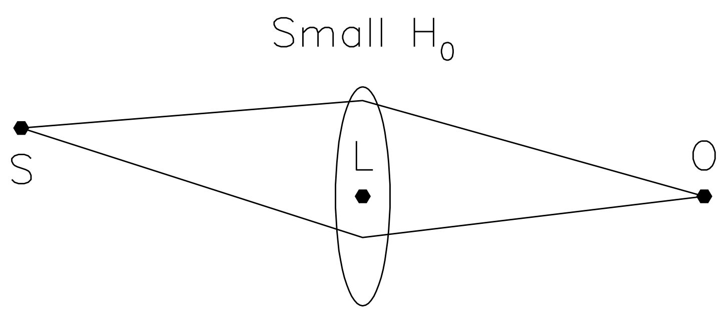
  e.g. $M=(1.08\pm0.02)h^{-1}*10^{10}$ Msol for QSO 2237+0305 within 0.9"

  Since the overall scale of the lens depends upon cosmological distances there is a dependence on H0.

H0 determination

Arrangement of images is purely geometric and contains no information about scales → Use time delay to learn about H0





$$
\begin{aligned}
t(\vec{\theta}) &= \frac{(1+z_{\rm d})}{c}\frac{D_{\rm d}D_{\rm s}}{D_{\rm ds}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2-\psi(\vec{\theta})\right] \\
&= t_{\rm geom}+t_{\rm grav}\ .
\end{aligned}
$$

$$D_A \propto H_0^{-1}$$

$$H_0 \Delta\tau = const$$

1. Use observed images to constrain model of lens to get constant
2. Measure time delay and get H0

Need multiple images and intrinsic variation in the source

In principle, just one very well understood lens enough to give a precision constraint on Ho

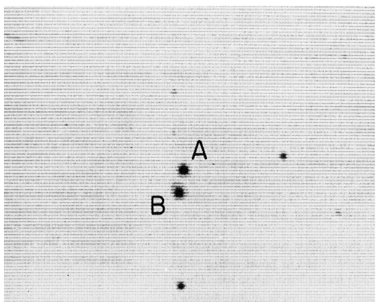
Totally independent of other measures of H0

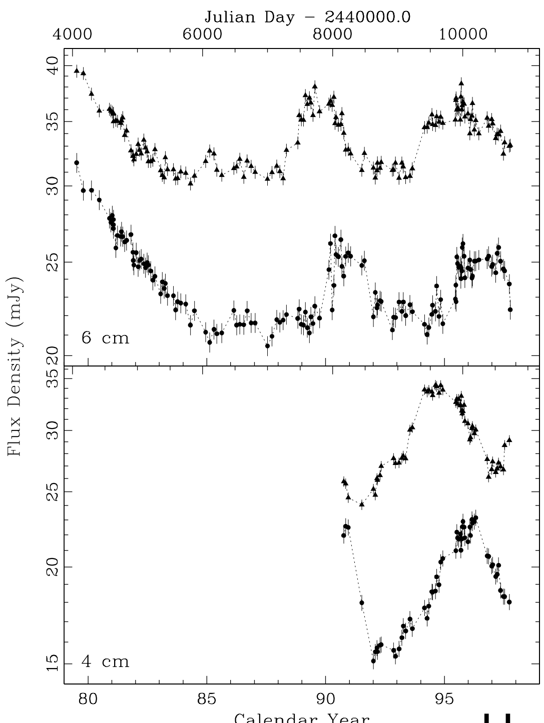
H0 determinations in practice

Radio time delays

QSO 0957+561

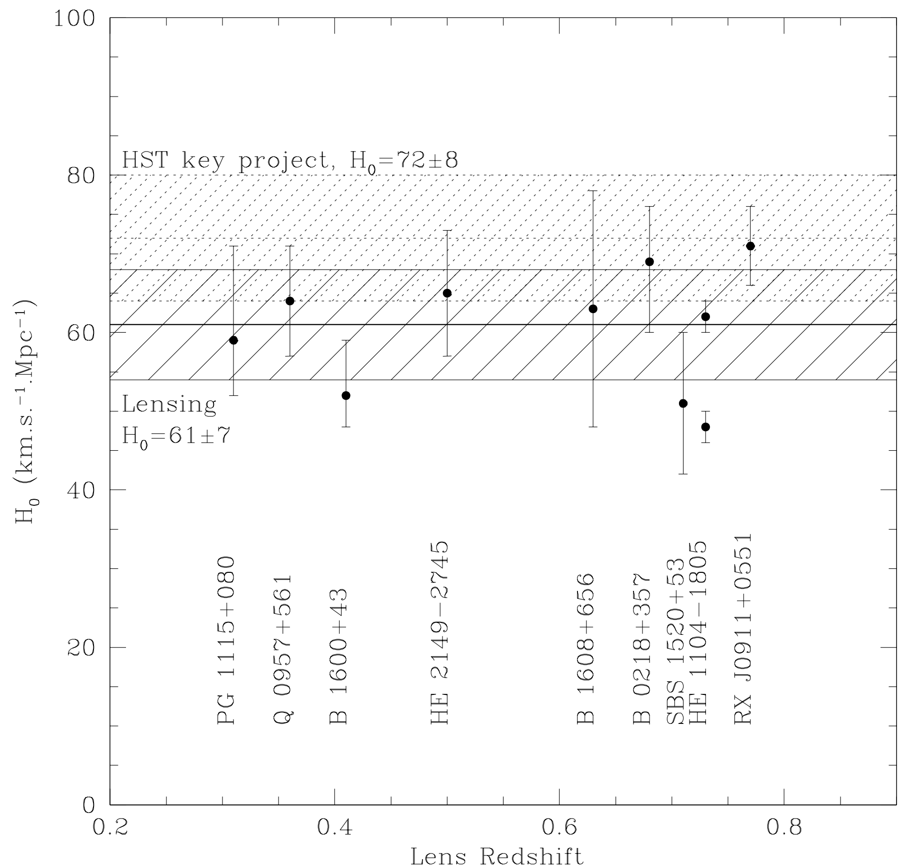
409 ± 30 days





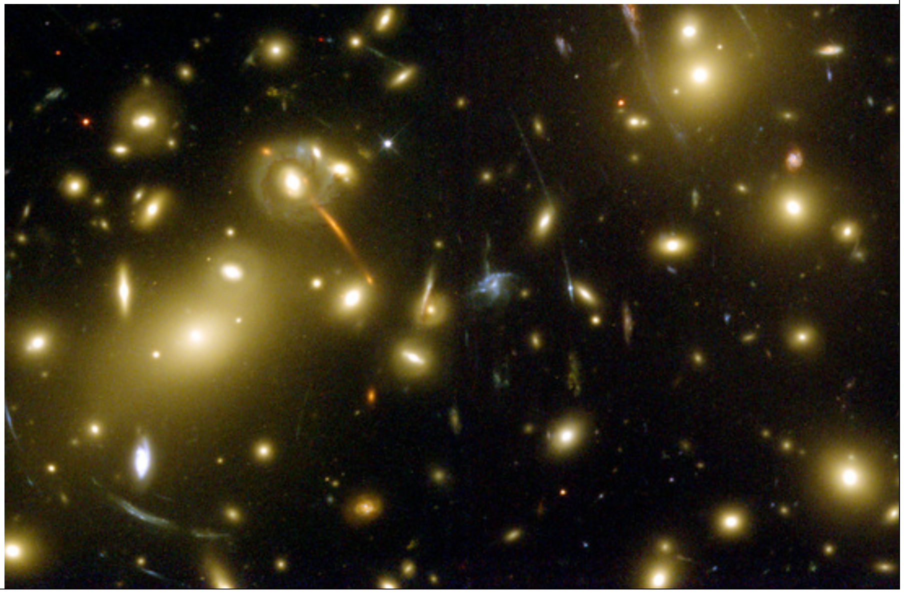
Haarsma 1999

Issues with modelling lens - limited constraints
Mass sheet degeneracy
Only ~10% quasars variable on useful timescales



Courbin 2003

Clusters



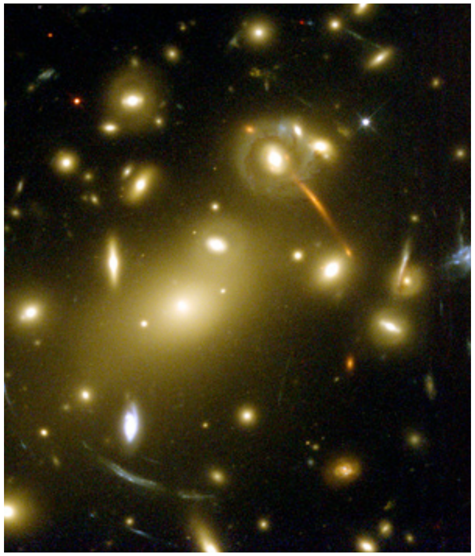
Cluster Masses

- We can take all of the theory that we've developed for galaxies and apply it to clusters. Again the isothermal sphere is a good starting point...
- Mass estimation assuming the arcs lie approximately on the Einstein radius

$$\langle \Sigma(\theta_{\rm arc}) \rangle \approx \langle \Sigma(\theta_{\rm E}) \rangle = \Sigma_{\rm cr}$$

$$M(\theta) = \Sigma_{\rm cr}\, \pi\, (D_{\rm d}\theta)^2 \approx 1.1 \times 10^{14}\, M_\odot \left(\frac{\theta}{30''}\right)^2 \left(\frac{D}{1\,{\rm Gpc}}\right)$$

- This agrees with estimates from X-ray luminosity estimates and dynamical estimates (a. la. Zwicky). Assumptions much simpler, so in principle this is a more accurate mass determination.
(caveat: really measuring mass inside a cylinder...)
- Often many background galaxies are multiply imaged by cluster - each can be used to constrain the mass distribution. Allows for constraints on cluster substructure
- Since the masses are much larger than that in the observed galaxies and gas in the cluster this is yet another indication of the presence of dark matter in the Universe.



Weak lensing

- So far we've focussed on the effects of strong lensing where the source passes in side the Einstein radius and multiple images are formed
- Although a weaker signal its also interesting to examine the effect on sources outside the Einstein radius, which are weakly lensed...
- By analogy to the giant arcs, we expect there to be smaller distortions to galaxies further from the center of a cluster (or a galaxy)
- Because the signal on an individual galaxy is smaller we need to average over many galaxies, which makes this a statistics based test
- Complicated by fact that galaxies are not intrinsically circular, but on average we don't expect the ellipticities of galaxies to have a preferred direction

Lensing potential

Introduce the lensing potential Ψ by projecting and rescaling the Newtonian potential

$$\psi(\vec{\theta}) = \frac{D_{\rm ds}}{D_{\rm d} D_{\rm s}} \frac{2}{c^2} \int \Phi(D_{\rm d}\vec{\theta}, z)\, dz$$

The gradient of this gives the deflection angle

$$\vec{\nabla}_\theta \psi = D_{\rm d} \vec{\nabla}_\xi \psi = \frac{2}{c^2} \frac{D_{\rm ds}}{D_{\rm s}} \int \vec{\nabla}_\perp \Phi\, dz = \vec{\alpha}$$

The divergence is similarly simply related to the surface mass density

$$\nabla^2_\theta \psi = \frac{2}{c^2} \frac{D_{\rm d} D_{\rm ds}}{D_{\rm s}} \int \nabla^2_\xi \Phi\, dz = \frac{2}{c^2} \frac{D_{\rm d} D_{\rm ds}}{D_{\rm s}} \cdot 4\pi G\, \Sigma = 2\, \frac{\Sigma(\vec{\theta})}{\Sigma_{\rm cr}} \equiv 2\kappa(\vec{\theta})$$

This gives us another way of thinking about the effects of lensing. We've seen that lensing will map an area in the source plane to an area in the image plane according to the lens equation. We can express this in terms of the matrix

$$\mathcal{A} \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left(\delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right) = \mathcal{M}^{-1}$$

Convergence

$$\kappa = \frac{1}{2}\left(\psi_{11} + \psi_{22}\right) = \frac{1}{2}\ \mathrm{tr}\ \psi_{ij}\ .$$

$$\frac{\partial^2 \psi}{\partial\theta_i \partial\theta_j} \equiv \psi_{ij}\ .$$

$$\kappa(\boldsymbol{\theta}) = \frac{\Sigma(\boldsymbol{\theta})}{\Sigma_{cr}}$$

$$\gamma = (\gamma_1^2 + \gamma_2^2)^{1/2}$$

Shear

$$\begin{aligned} \gamma_1(\vec{\theta}) &= \frac{1}{2}\left(\psi_{11} - \psi_{22}\right) \equiv \gamma(\vec{\theta}) \cos\left[2\phi(\vec{\theta})\right]\ , \\ \gamma_2(\vec{\theta}) &= \psi_{12} = \psi_{21} \equiv \gamma(\vec{\theta}) \sin\left[2\phi(\vec{\theta})\right]\ . \end{aligned}$$

Rewrite transformation matrix

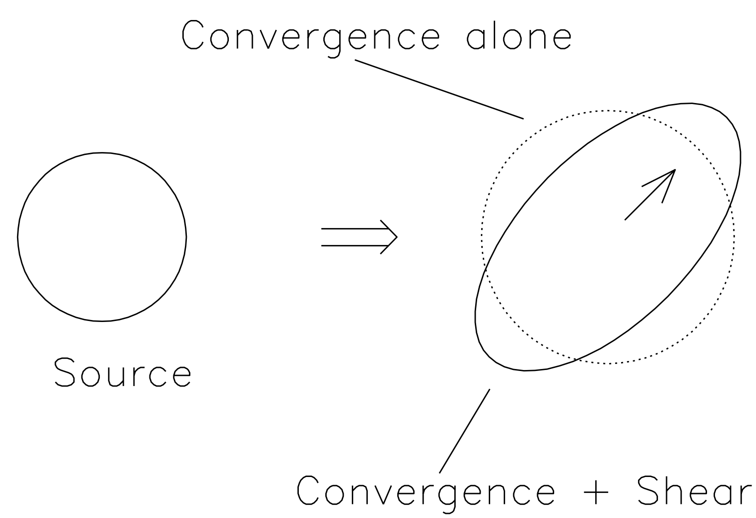
$$\begin{aligned} \mathcal{A} &= \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \\ &= (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \end{aligned}$$

Magnification

$$\mu = \det \mathcal{M} = \frac{1}{\det \mathcal{A}} = \frac{1}{[(1-\kappa)^2 - \gamma^2]}\ .$$

Major and minor axes

$$(1-\kappa-\gamma)^{-1}\ , \quad (1-\kappa+\gamma)^{-1}$$



Using ellipticity to infer mass density

- Both convergence and sheer determined by same lensing potential
- Measure ellipticities of galaxies and use to infer the shear field
- Infer lensing potential from shear
- Obtain convergence and so mass density from divergence of lensing potential

Shear maps

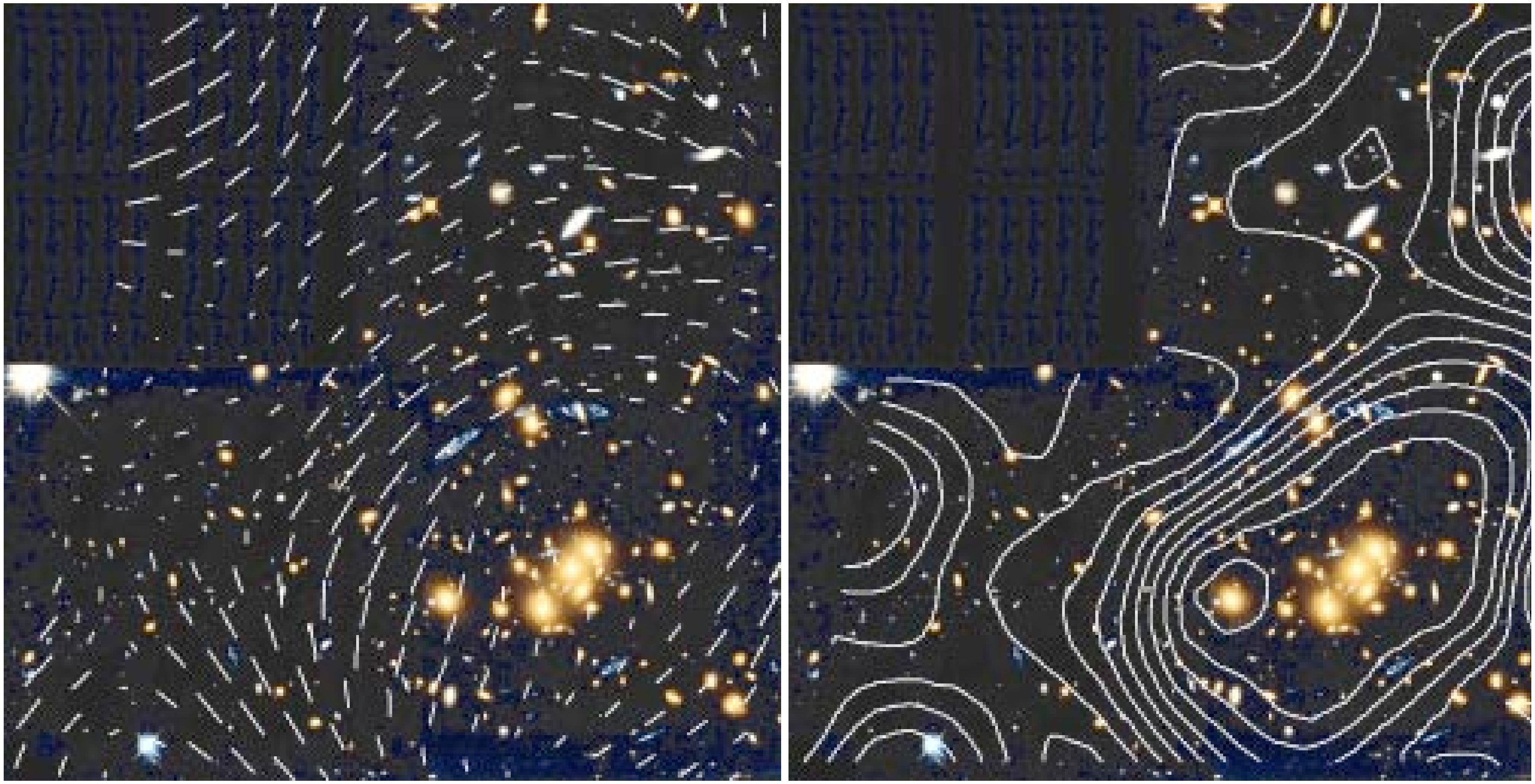
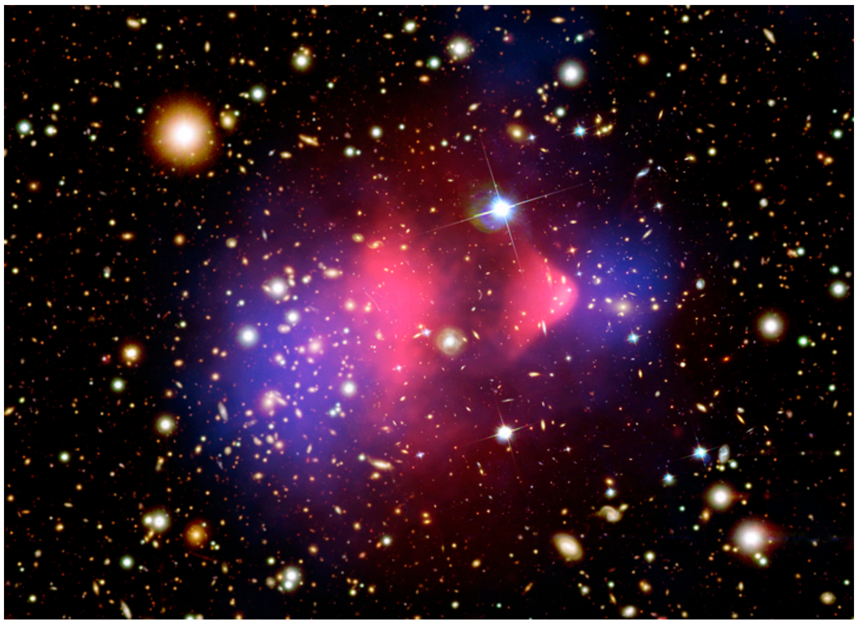


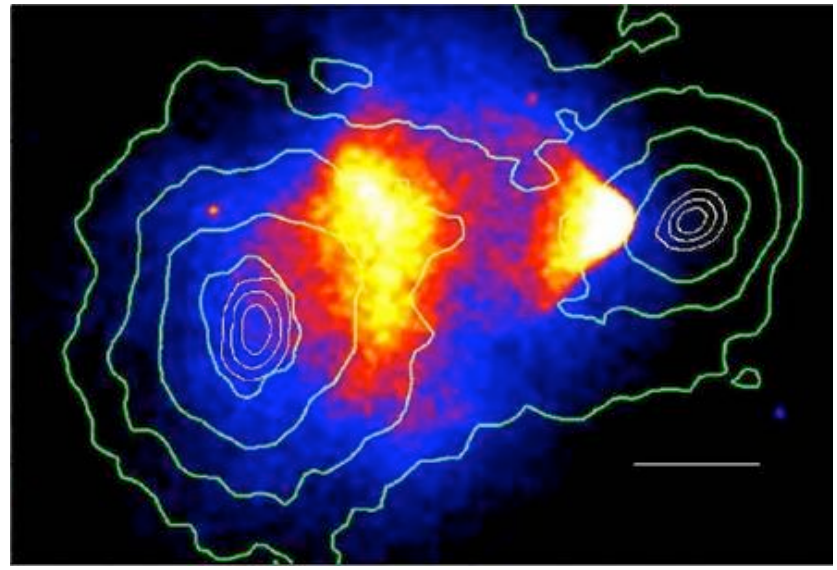
FIG. 25.—*HST* image of the cluster Cl 0024, overlaid on the left with the shear field obtained from an observation of arclets with the *Can*
France Hawaii Telescope (Y. Mellier & B. Fort), and on the right with the reconstructed surface-mass density determined from the shear fiel
Seitz et al.). The reconstruction was done with a non-linear, finite-field algorithm.

Generally centers of light and mass coincide
as do centers of X-ray emission and mass

Bullet Cluster



HST: galaxy distribution
Chandra: X-ray map



HST: mass contours from weak lensing
Chandra: X-ray map

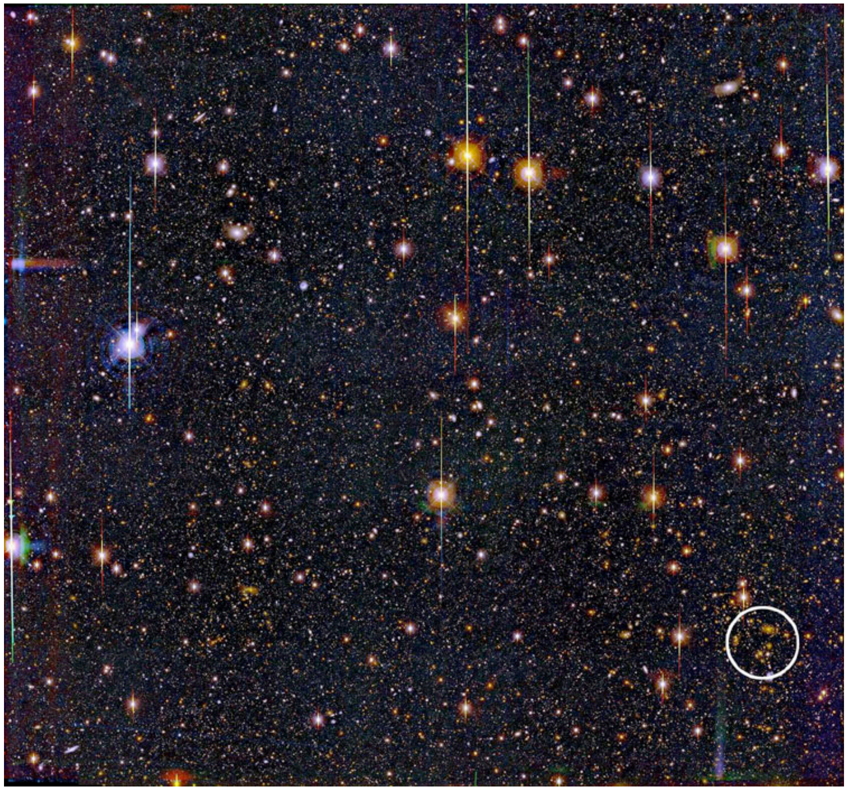
Clowe+ 2006

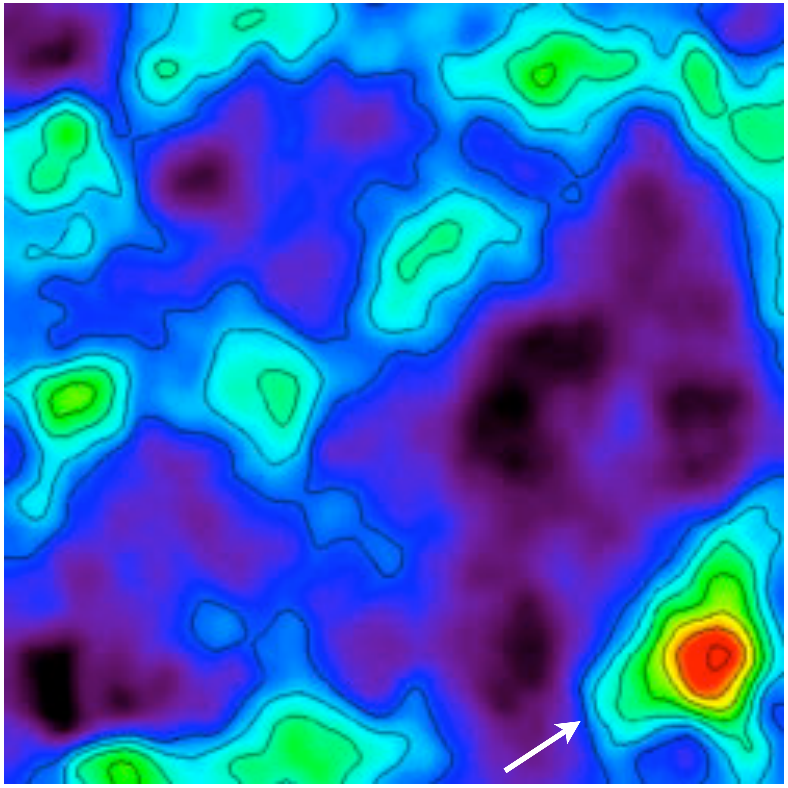
Weak+strong lensing shows that mass center and majority of the matter do not occur at the same location

Cluster searches

Can use weak lensing to find new clusters by looking for peaks in the inferred mass distribution - solely based on mass critera not emitted radiation
- complementary to current SZ searches

Potential probe of clusters with low gas or galaxy content if they exist "dark clusters"



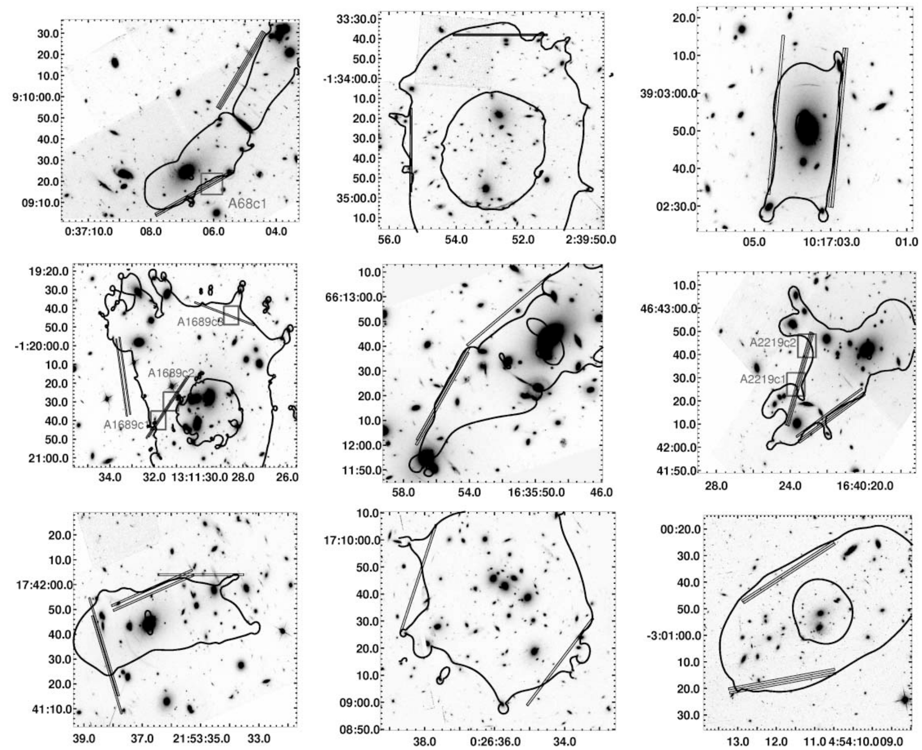


Clusters as gravitational telescopes

Use gravitational lensing to help find faint galaxies at high redshift (z>7)

Significant magnification (>20) occurs along the critical lines of clusters enabling faint galaxies to be detected

By focusing their search along the known critical curves of nine clusters. Stark et al (2007) were able to find some of the highest redshift galaxies yet discovered. This provides one of our only views onto the sources responsible for reionization.

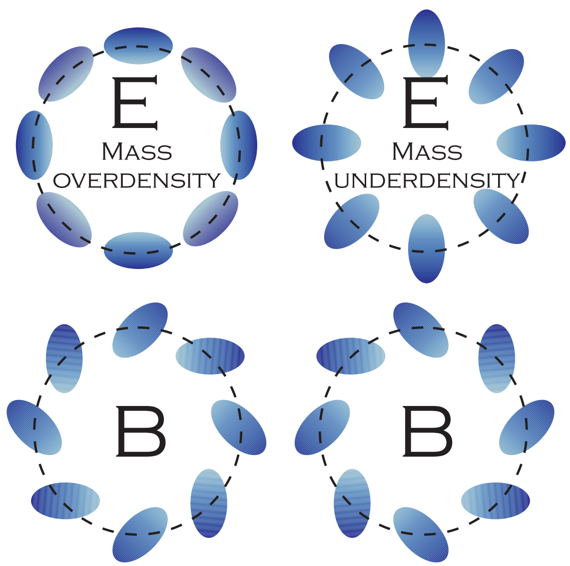


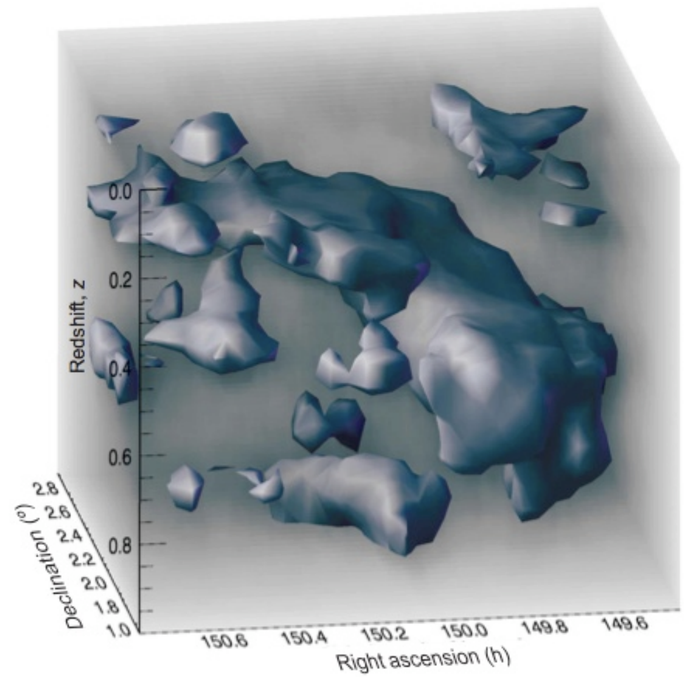
Stark+ 2007

Trade magnification for small survey volume in source plane

Cosmic Shear and LSS

- In the coming decade, there will be a number of large area, deep, galaxy surveys such as JDEM, LSST, etc.
- By measuring the shape of millions of galaxies across the whole sky one can hope to measure the density distribution of large scale structure. This approach has enormous power for cosmology.
- 3D mapping of DM achieved by using source populations at different redshifts
- Control systematics using E/B decomposition





Massey+ 2007 (COSMOS)

Cosmic sheer in cosmology

- Probe modifications to gravity via growth of structure
- Some claimed inconsistencies between growth of structure predicted using GR and weak lensing data (Bean 2009)
- Weak lensing also important for CMB: lensed E mode polarization looks like B mode polarization and conceals the B-mode signature of inflation from gravitational waves

The end

- Normal service will resume next lecture